Structural Model for Preconditioning in Ligaments and Tendons

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  - Preconditioning
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- Results
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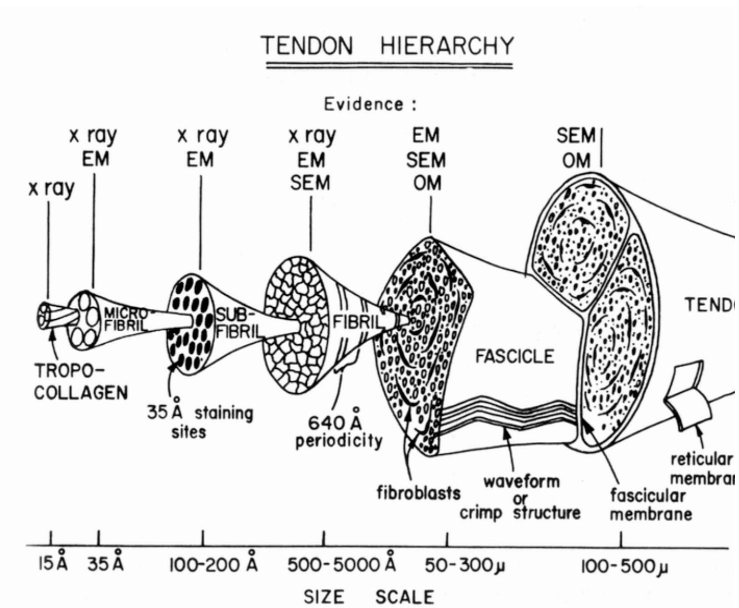
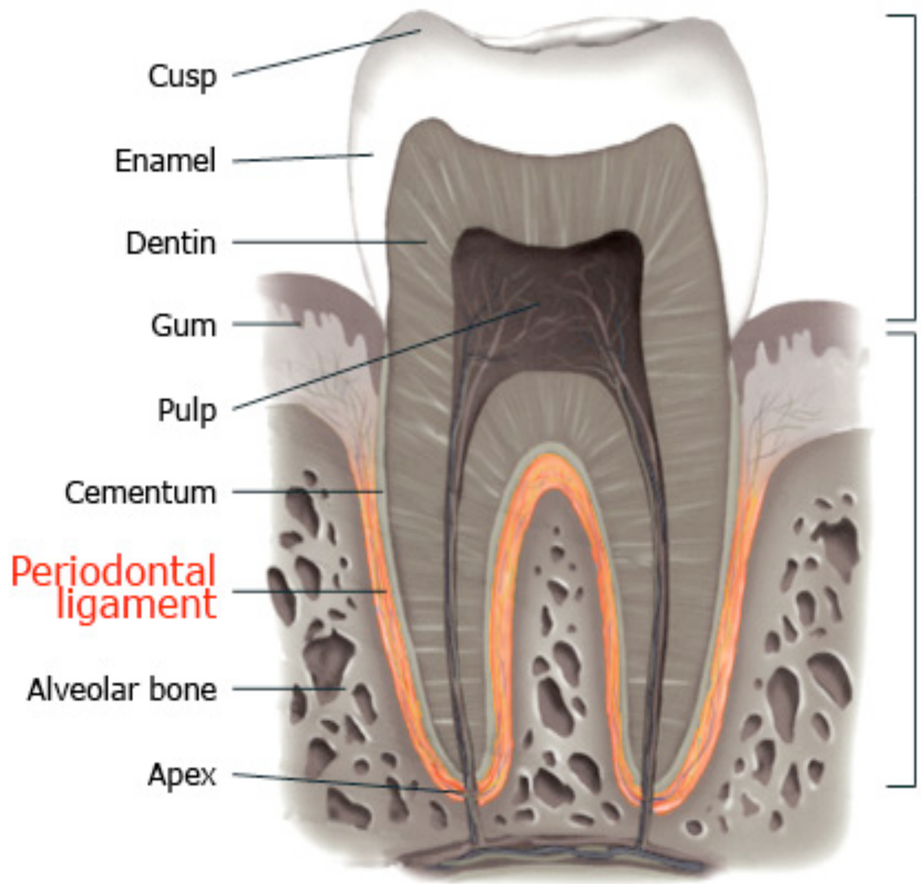
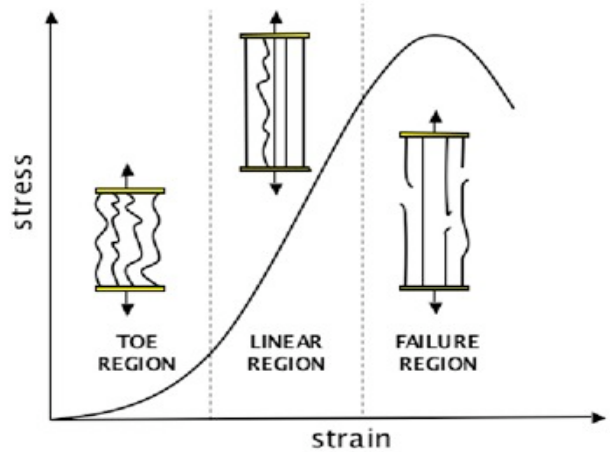
Ligaments and Tendons: Structure

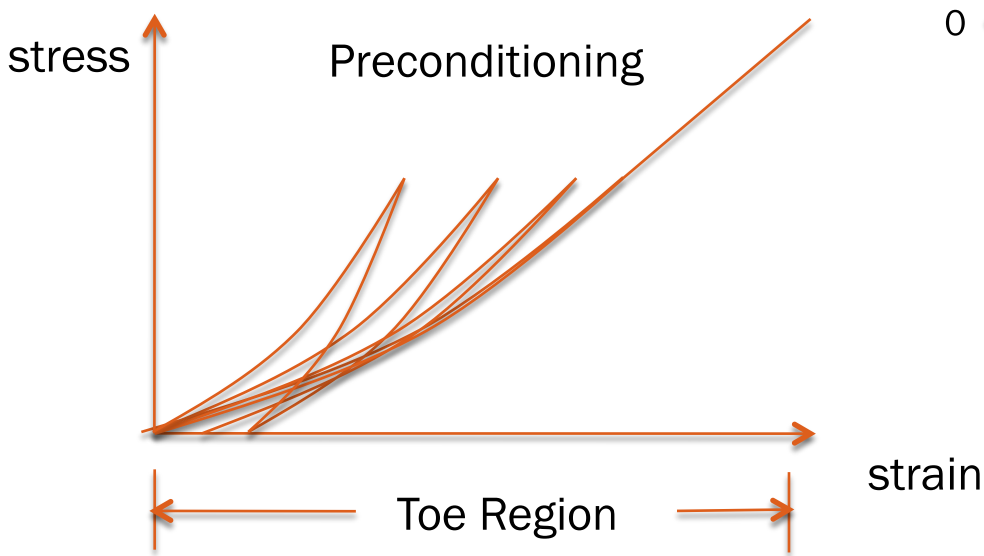


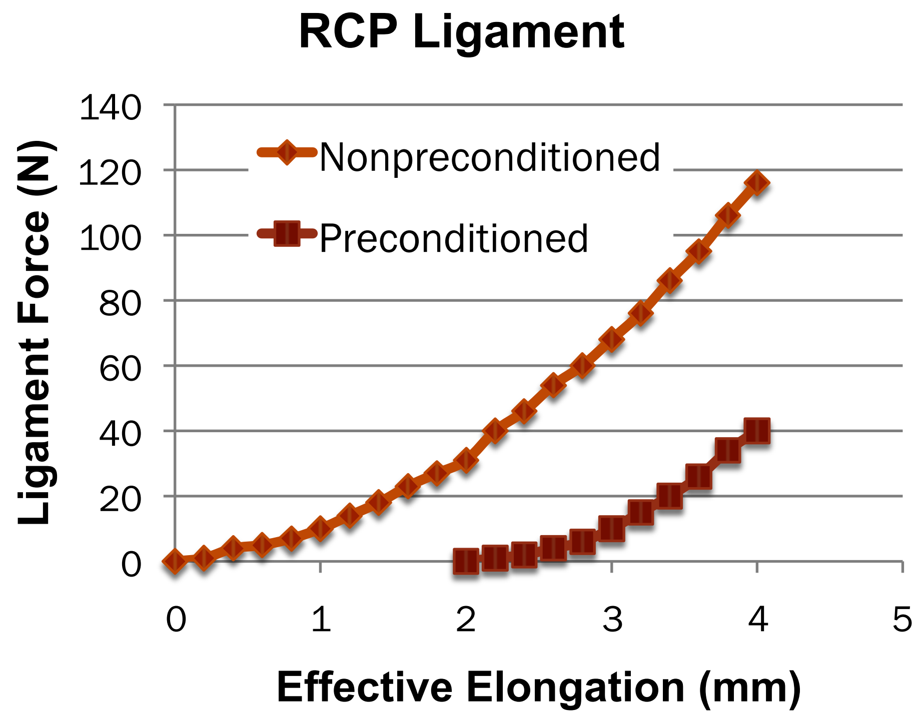


Tensile Behavior



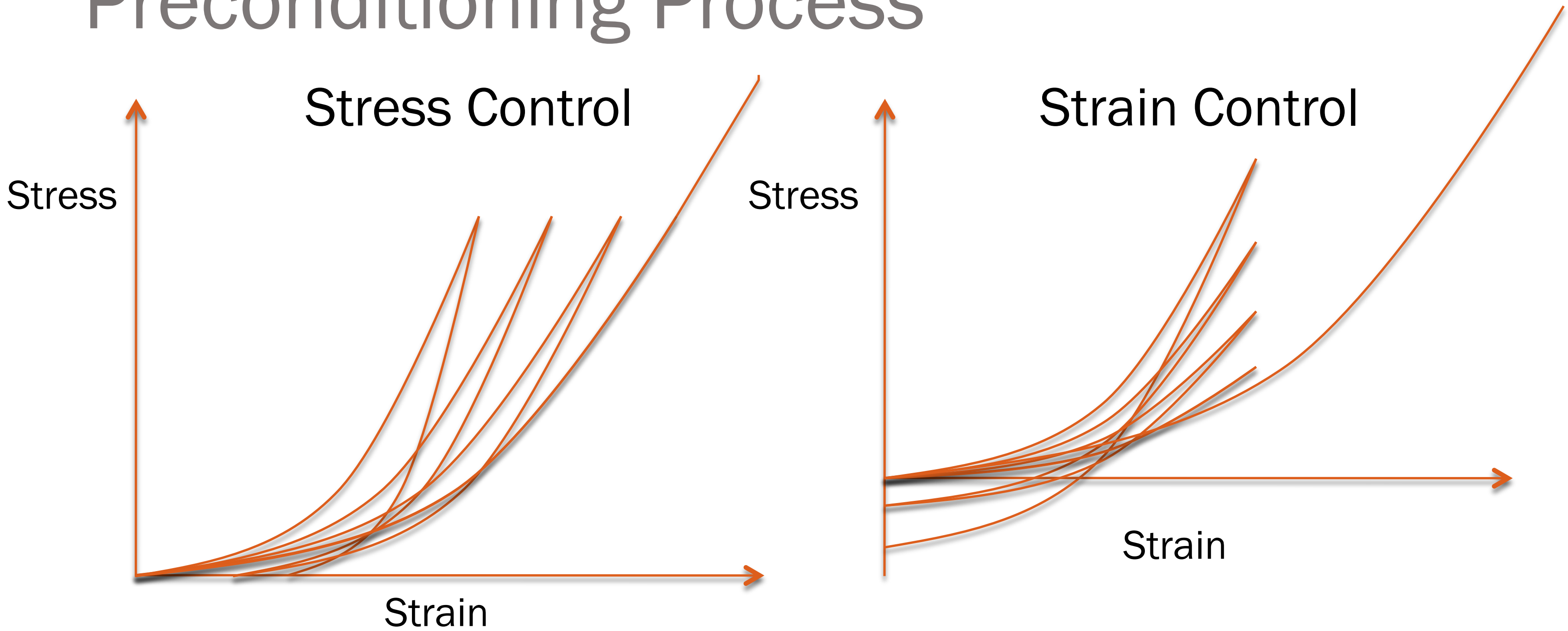
Typical stress – strain curve





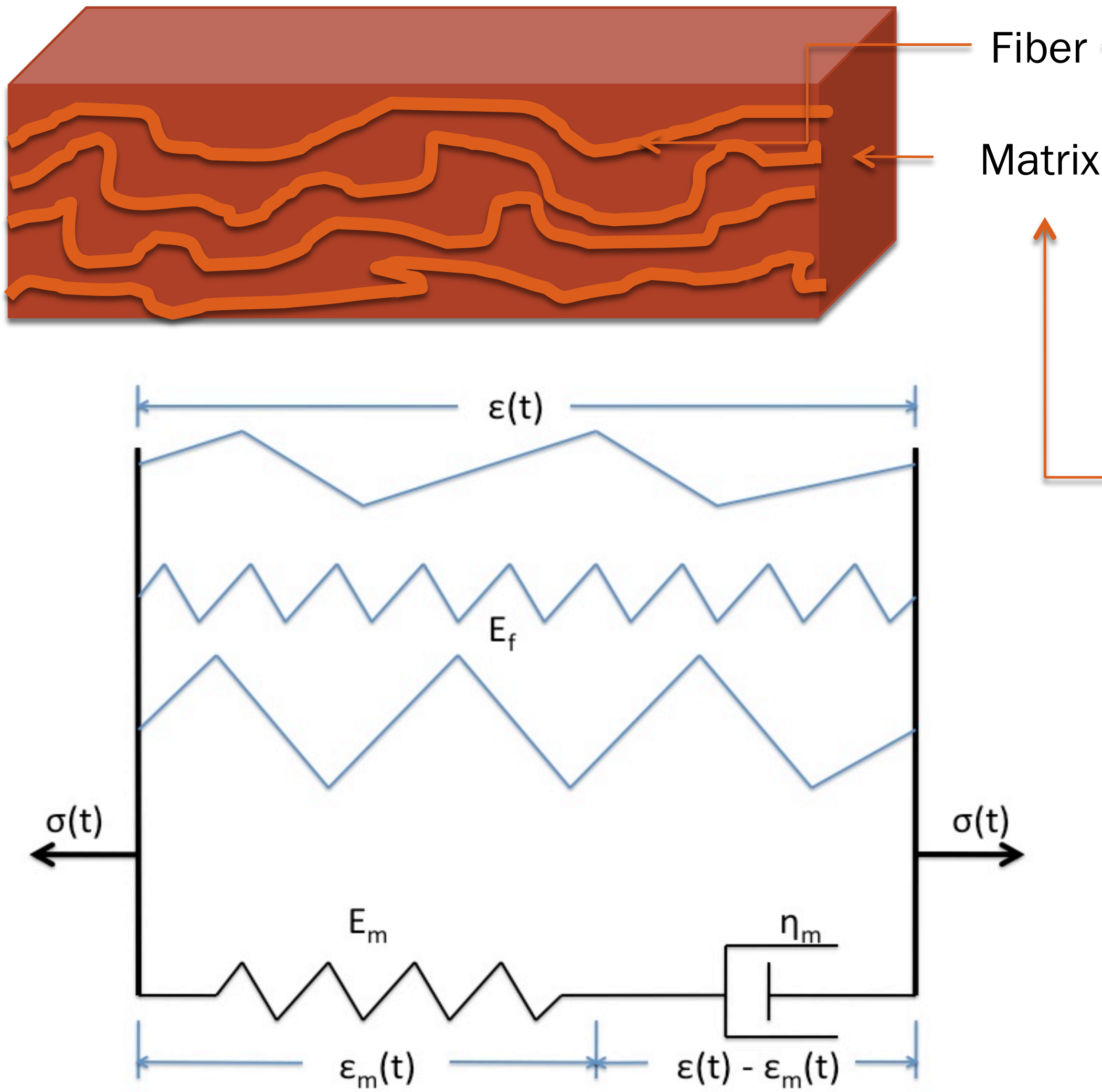
The comparison of the experimental data between before preconditioning and after preconditioning (Savelburg et al., 1993)

Preconditioning Process



- Tissue's internal structure changes with (stress controlled or strain controlled) cycling loading.
- At the steady state, no further change occurs (unless cycling routine changes).

Model Formulation



$E_f \equiv$ Fiber Stiffness
$\sigma_f \equiv$ Total Fibers Stress

+

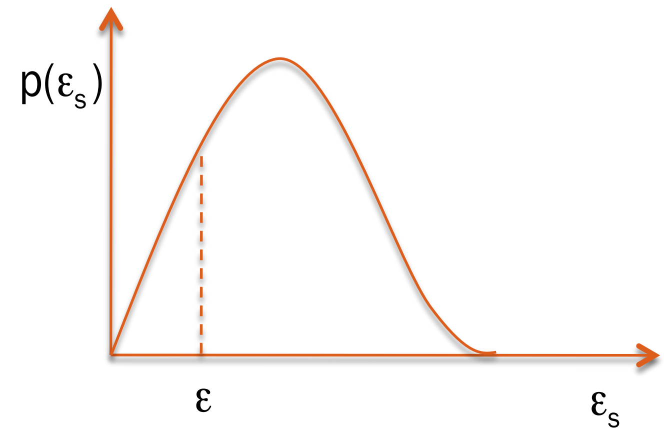
$E_m \equiv$ Matrix Stiffness
$\varepsilon_m \equiv$ Matrix Elastic Strain
$\sigma_{E_m} \equiv$ Matrix Elastic Stress

$\eta_m \equiv$ Matrix Viscosity
$\sigma_{\eta_m} \equiv$ Matrix Viscous Stress

$\varepsilon(t) \equiv$ Total Tissue Strain
$\sigma(t) \equiv$ Total Tissue Stress

Total Fibers Stress:

$$\sigma_f(t) = \int_0^{\varepsilon} E_f(\varepsilon - \varepsilon_s)p(\varepsilon_s)d\varepsilon_s$$



ε_s = straight fiber strain

Weibull Distribution:

$$p(\varepsilon_s) = \frac{\alpha}{\beta}\left(\frac{\varepsilon_s}{\beta}\right)^{\alpha-1} e^{-\left(\frac{\varepsilon_s}{\beta}\right)^{\alpha}}, \quad \varepsilon_s \geq 0$$

Fraction of straight fibers during loading:

$$x(\varepsilon_s) = \int_0^{\varepsilon} p(\varepsilon_s)d\varepsilon_s$$

Stress Equilibrium:

$$\sigma(t) = \sigma_f(t) + \sigma_{E_m}(t) = \sigma_f(t) + \sigma_{\eta_m}(t)$$

$$\sigma_{E_m}(t) = E_m \varepsilon_m(t)$$

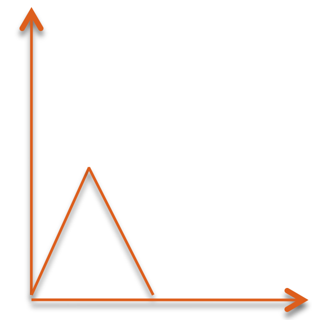
$$\sigma_{\eta_m}(t) = \eta_m \frac{d}{dt}(\varepsilon(t) - \varepsilon_m(t))$$

Governing Equation:

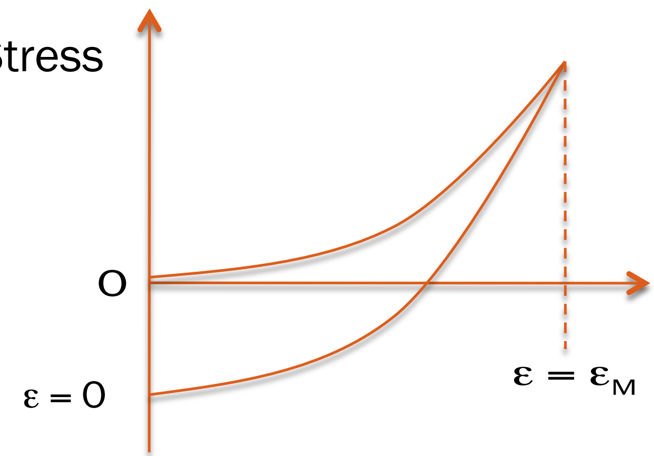
$$\sigma'(t) + \frac{E_m}{\eta_m}\sigma(t) = \sigma_f'(t) + \frac{E_m}{\eta_m}\sigma_f(t) + E_m \varepsilon'(t)$$

General Solution:

$$\sigma(t) = \sigma_f(t) + \frac{\int E_m \varepsilon'(t) e^{t/\tau} dt + C}{e^{t/\tau}}$$



where $\tau = \frac{\eta_m}{E_m}$,C = constant determined by initial condition



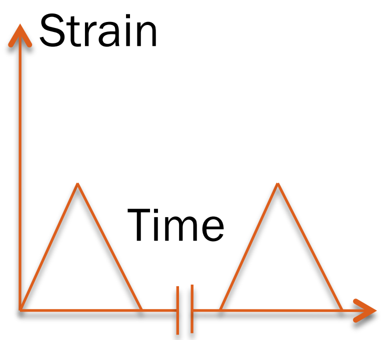
Softening Model:

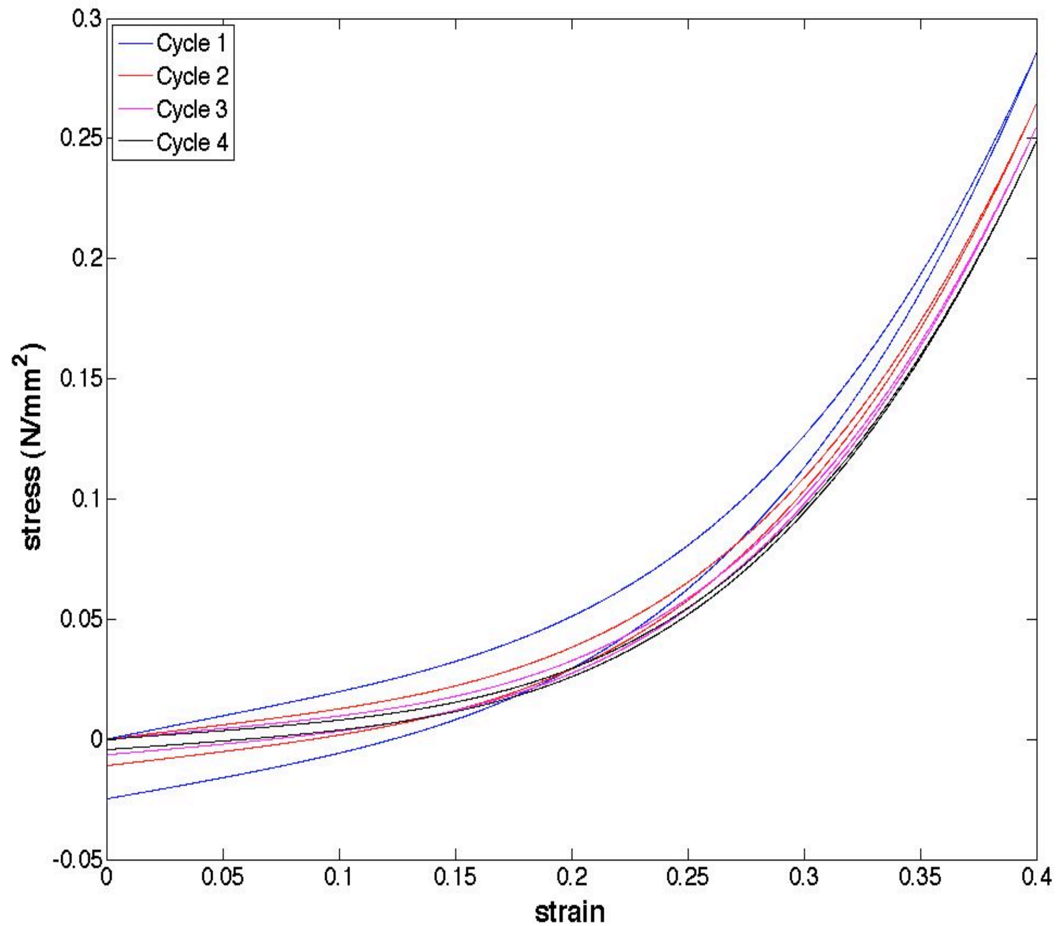
$$E_m^{i+1} = \left(1 - \frac{\varepsilon^i_{m@\varepsilon=0}}{\varepsilon^i_{m@\varepsilon=\varepsilon_M}}\right) E^i_m$$

where i = the number of loop, $\varepsilon_{m@\varepsilon=0} \geq 0$

Preliminary Results

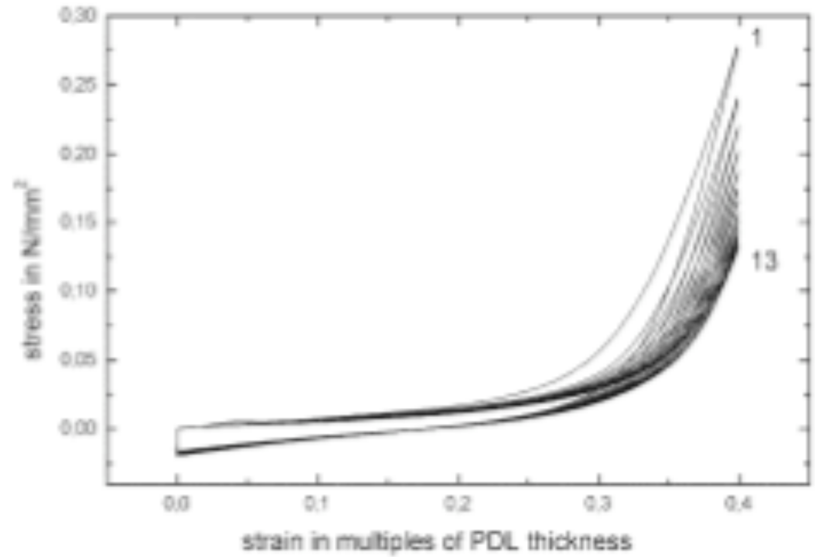
Cyclic Strain Input:



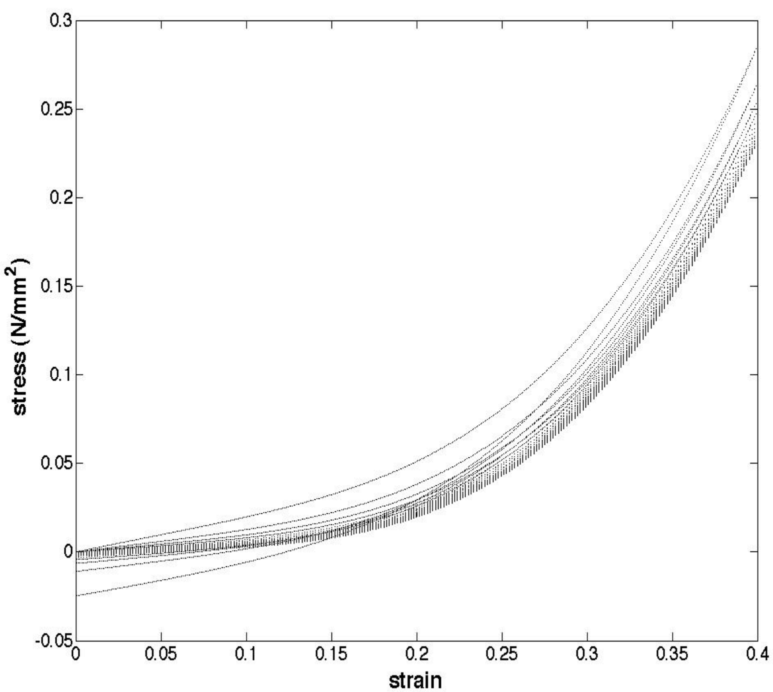


$$\alpha = 3, \beta = 0.5, E_f = 5\text{MPa}, E_m = 0.2\text{MPa}, \tau = 1\text{s}^{-1}, \varepsilon' = 0.8\text{s}^{-1}$$

Preliminary Results



The experimental data from Dorow et al. (Dorow et al., 2001)



$$\alpha = 3, \beta = 0.5, E_f = 5\text{MPa}, E_m = 0.2\text{MPa}, \tau = 1s^{-1}, \varepsilon' = 0.8s^{-1}$$

Conclusion

- The model can physically describe preconditioning in ligaments and tendons by accounting for their internal structure.
- These preliminary simulations have good qualitative agreement with experimental data published by Dorow et al. (2001).
- Evaluation of model parameters requires more experimental data.
- The model will be extended to describe other viscoelastic behaviors (e.g. relaxation and creep).