A Probabilistic Constitutive Law For Damage in Ligaments

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Outline

- Introduction
  - Ligament Functions and Injuries
  - Ligament Morphology
  - Ligament Tensile and Damage Behavior
- Constitutive Model
  - Assumptions
  - Model Formulation
  - Stress-Stretch Relationship
- Results
  - Model Fitting Results
  - Predictions for Different Subfailure Stretches
  - Effects of Model Parameters
- Conclusions

Introduction

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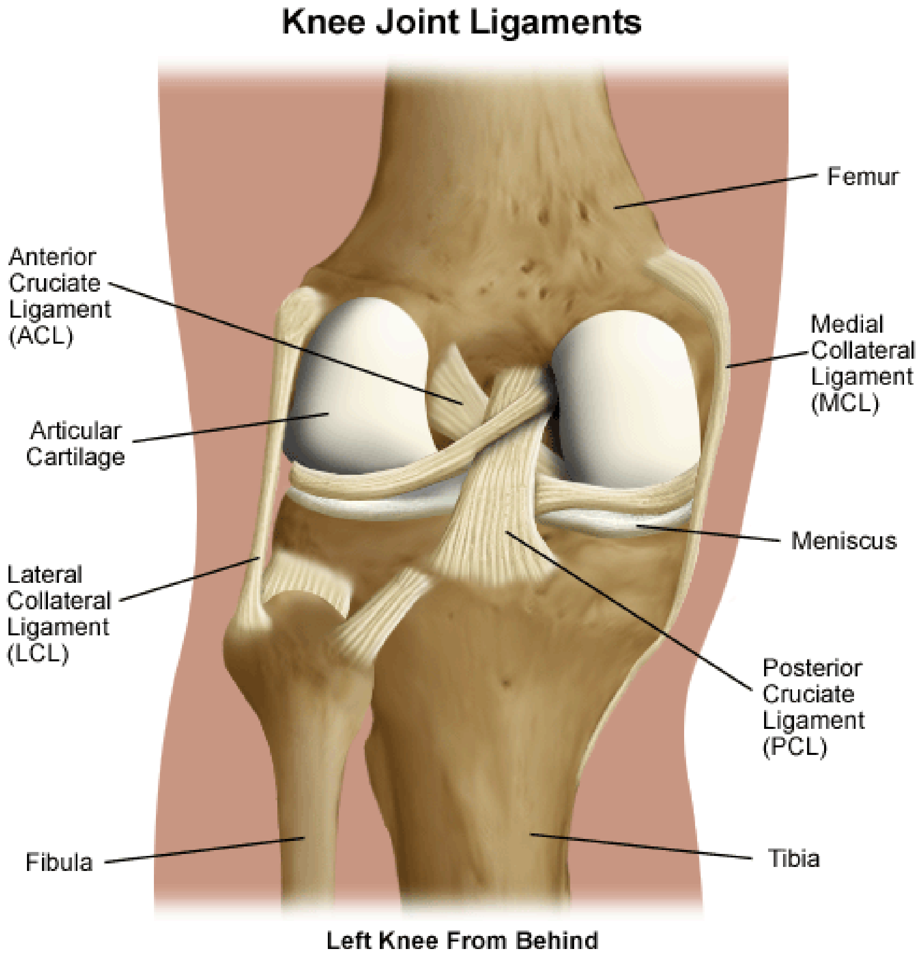
Ligament Functions and Injuries

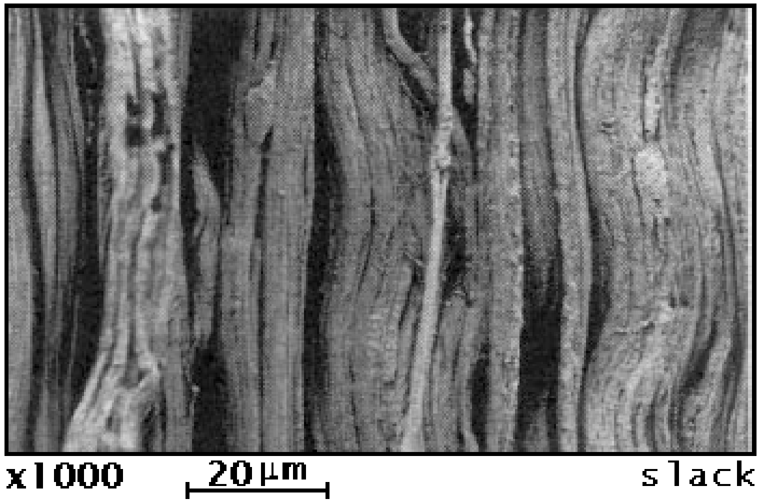


- Ligament functions
  - Connect bone to bone
  - Guide and restrain joint motions
- Ligament injuries
  - Most commonly injured ligaments in sports are ACLs, MCLs (90% of knee ligament injuries)
  - 85% of the ligament injuries consist of first degree and second degree sprains

Ligament Morphology

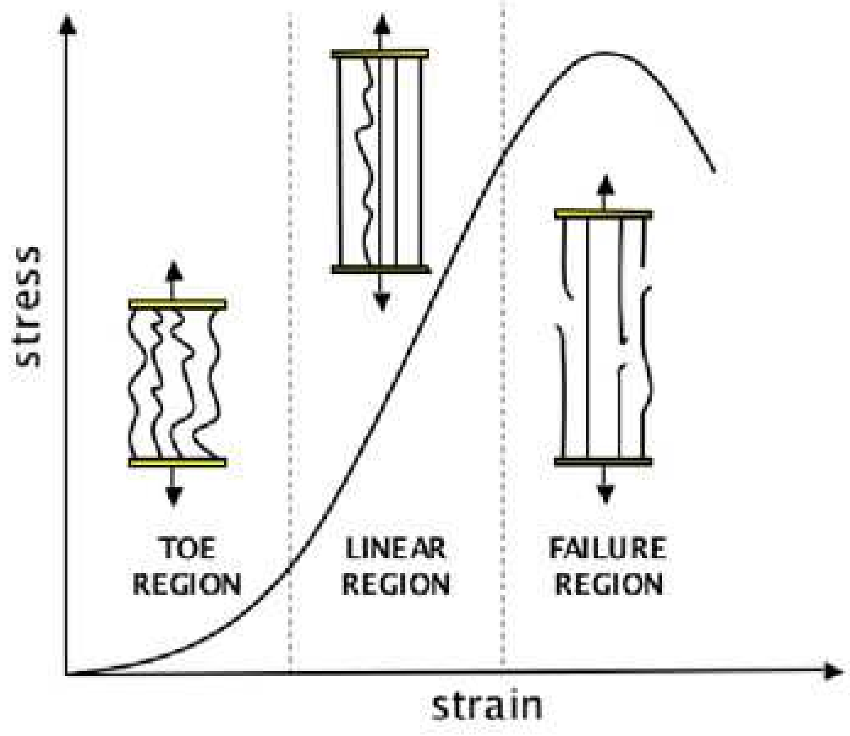
- Components:
  - Collagen fibers (70%~80% of dry weight)
  - Ground substance
    - Water (65%~70% of the total weight)
    - Proteoglycans, glycoproteins
  - Elastin (1%~2% of dry weight)

Collagen fiber waviness

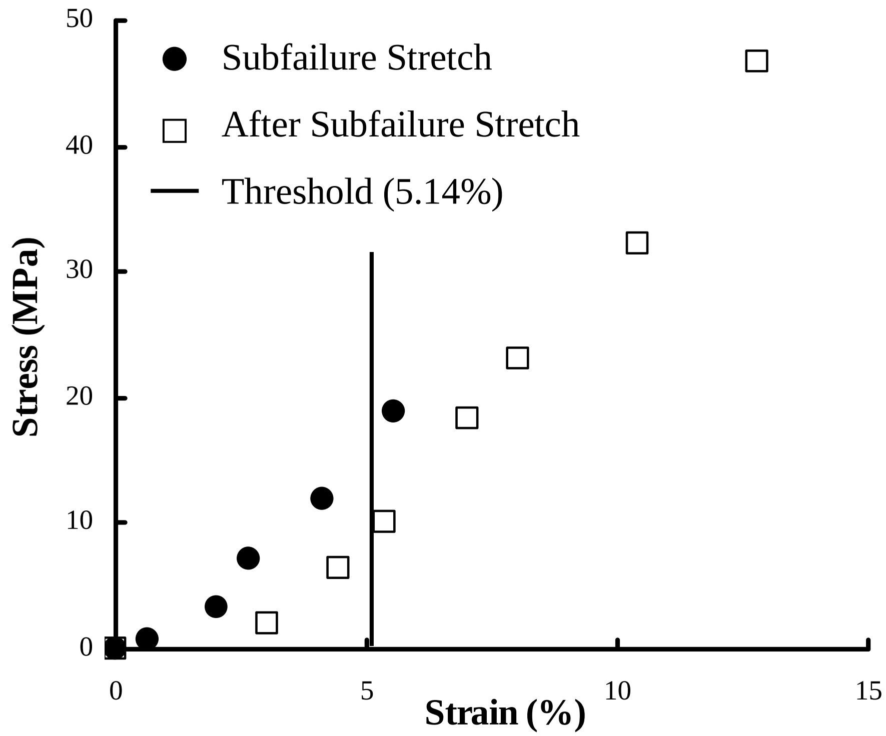


SEM of Rat MCL
(Provenzano, 2002)

Ligament Tensile and Damage Behavior



Stress-strain relation



Stress-strain relation of rat MCL after subfailure stretch

(Provenzano, 2002)

Model Formulation

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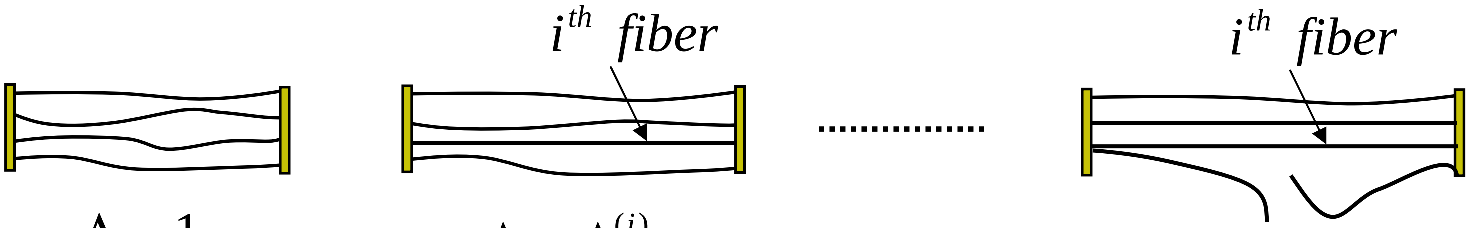
Model Assumptions

- Ligament tensile behavior is determined only by collagen fibers.
- Fibers are parallel and aligned along ligament physiological loading direction.
- Fibers become straight at different stretches. Straight fiber behaves like a spring.
- Damage process is controlled by the stretch.
- Damage is caused by the failure of the fibrils that make up the collagen fiber.
- At each damage stretch, one fibril breaks and the stiffness of the fiber reduces by a damage factor.

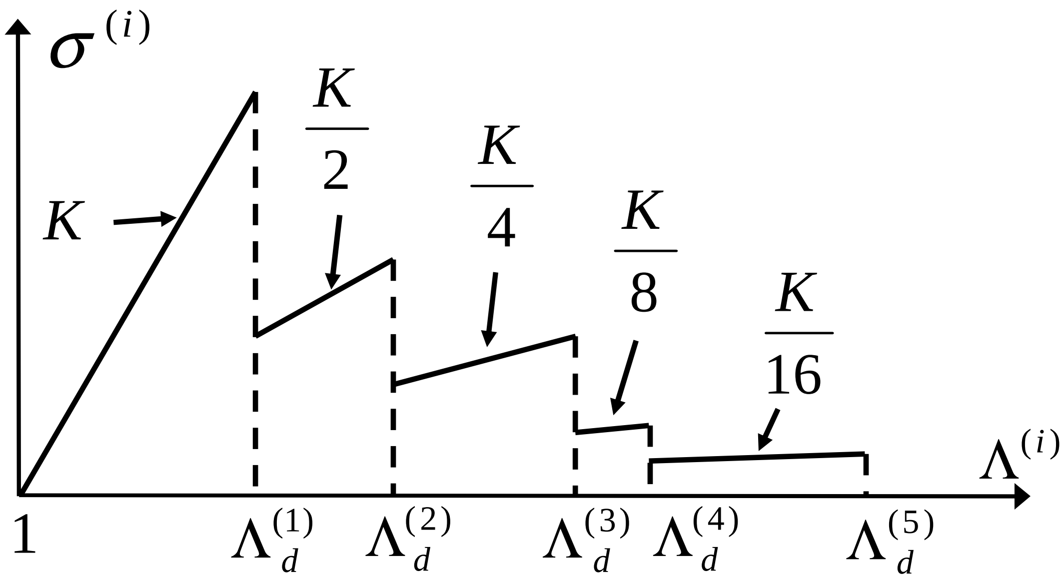
Model Schematic



Ligament stretch: $\Lambda = 1$ $\quad\quad$ $\Lambda = \Lambda_s^{(i)}$

i^{th} fiber stretch: $\Lambda^{(i)} = \Lambda / \Lambda_s^{(i)} = 1$ $\quad\quad$ $\Lambda^{(i)} = \Lambda_d^{(j)}$

Damage process for i^{th} fiber with five damage stretches. The stiffness reduction factor D=0.5



Model Formulation

- Generate N fibers with straightening stretches $\Lambda_s^{(i)}$, which are randomly defined by a Weibull distribution:

$$\Lambda_s^{(i)}(P_s^{(i)},\alpha_s,\beta_s,\gamma_s)=\gamma_s+\beta_s[-\ln(1-P_s^{(i)})]^{\frac{1}{\alpha_s}}$$

  - i =1 … N (integer)
  - $0<P_s^{(i)}<1$: random number
  - $\alpha_s>0$: shape parameter
  - $\beta_s>0$: scale parameter
  - $\gamma_s=1$: location parameter

Model Formulation (con't)

- A single straight fiber has M damage stretches. The damage stretches $\Lambda_d^{(j)}$ are randomly generated by another Weibull distribution:

$$\Lambda_d^{(j)}(P_d^{(j)}, \alpha_d, \beta_d, \gamma_d) = \gamma_d + \beta_d[-\ln(1 - P_d^{(j)})]^{\frac{1}{\alpha_d}}$$

  - j=1 … M (integer)
  - $0 < P_d^{(j)} < 1$: random number
  - $\alpha_d > 0$: shape parameter
  - $\beta_d > 0$: scale parameter
  - $\gamma_d = 1.0514$: location parameter

Stress-Stretch Relation

- Stress of a single straight fiber

$$\sigma^{(i)} = \begin{cases} 0 & \Lambda^{(i)} \leq 1 \\ K(\Lambda^{(i)} - 1) & 1 < \Lambda^{(i)} < \Lambda_d{}^{(j)} \\ D^k K(\Lambda^{(i)} - 1) & \Lambda^{(i)} \geq \Lambda_d{}^{(k)}, (\Lambda_d{}^{(k)} > \Lambda_d{}^{(k-1)} > \cdots > \Lambda_d{}^{(1)}) \end{cases}$$

  - Λ : ligament stretch
  - $\Lambda^{(i)} = \Lambda / \Lambda_s^{(i)}$: fiber stretch relative to straight configuration
  - K : fiber's elastic modulus
  - $0 < D < 1$: damage reduction factor

- Ligament stress

$$\sigma = \frac{1}{N} \sum_{i=1}^{N} \sigma^{(i)}$$

Results

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Model Fitting Result

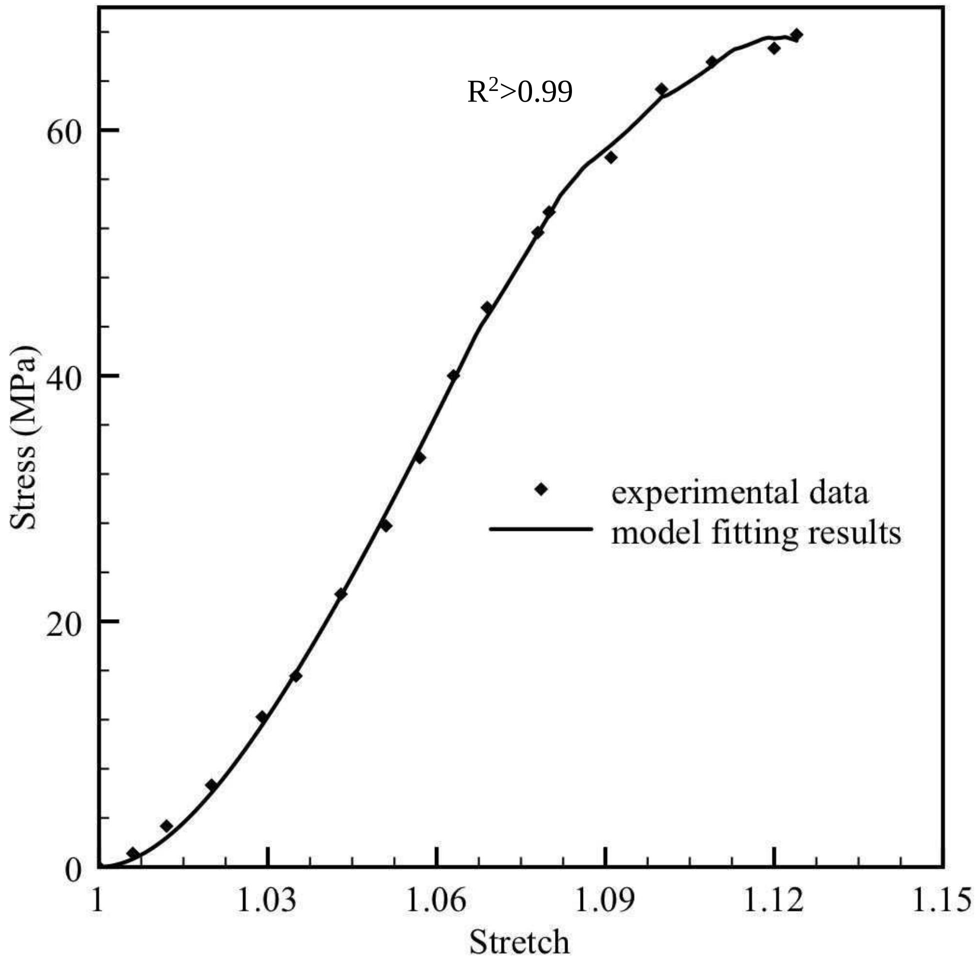

Figure 1. Fitting to rat MCL tensile experimental data (Provenzano, 2002)

Fix M=100, N =100,000

4 Weibull distribution parameters :

$\{\alpha_s, \beta_s, \alpha_d, \beta_d\}$

2 material parameters :

$\{K, D\}$

Parameters	
K	: 1059 MPa
α_s	: 1.000
β_s	: 0.02727
α_d	: 2.052
β_d	: 0.2357
D	: 0.8992

Results: Effect of Different Subfailure Stretches

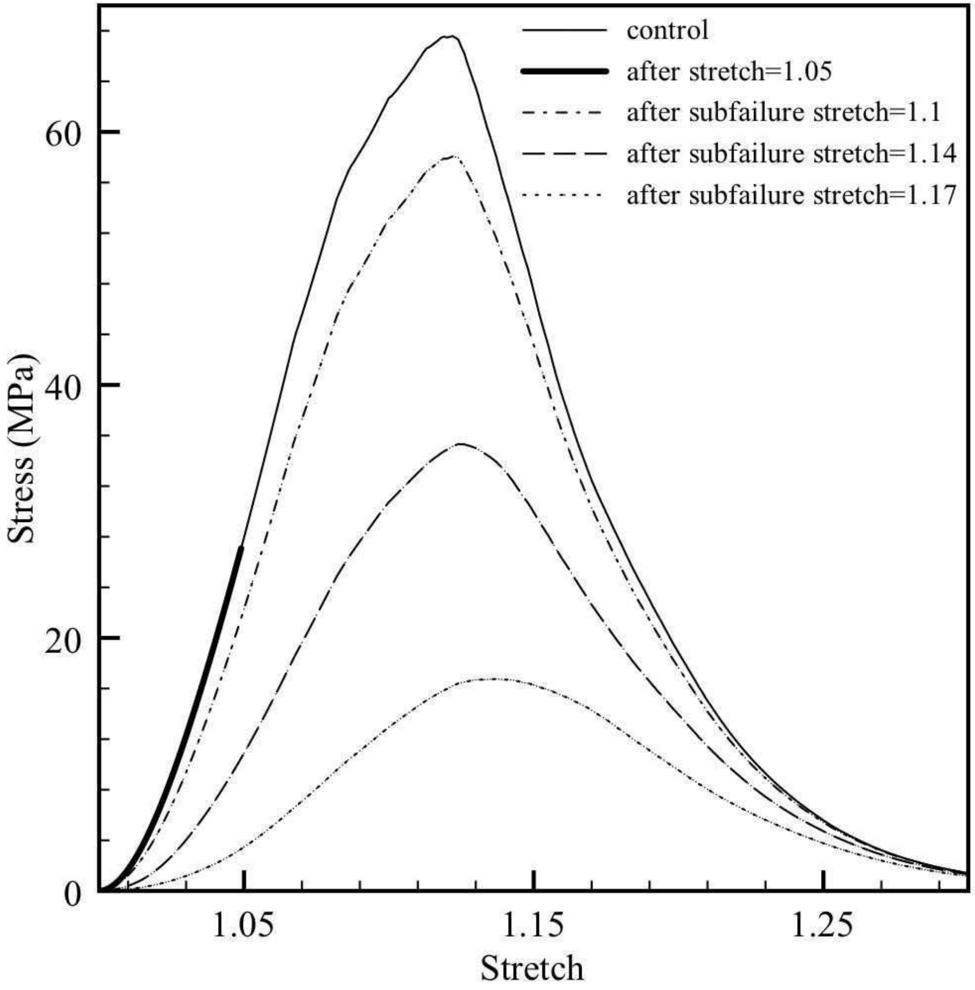

Figure 2. Model prediction of different subfailure stretches

Results: Varying Straightening Parameters

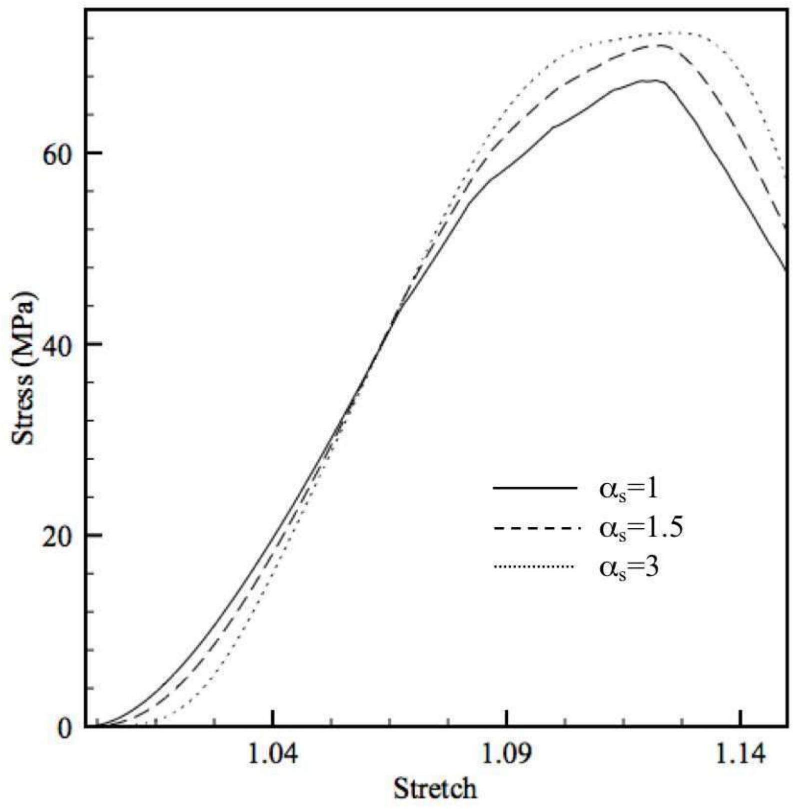

Figure 3. Model predictions of different α_s values

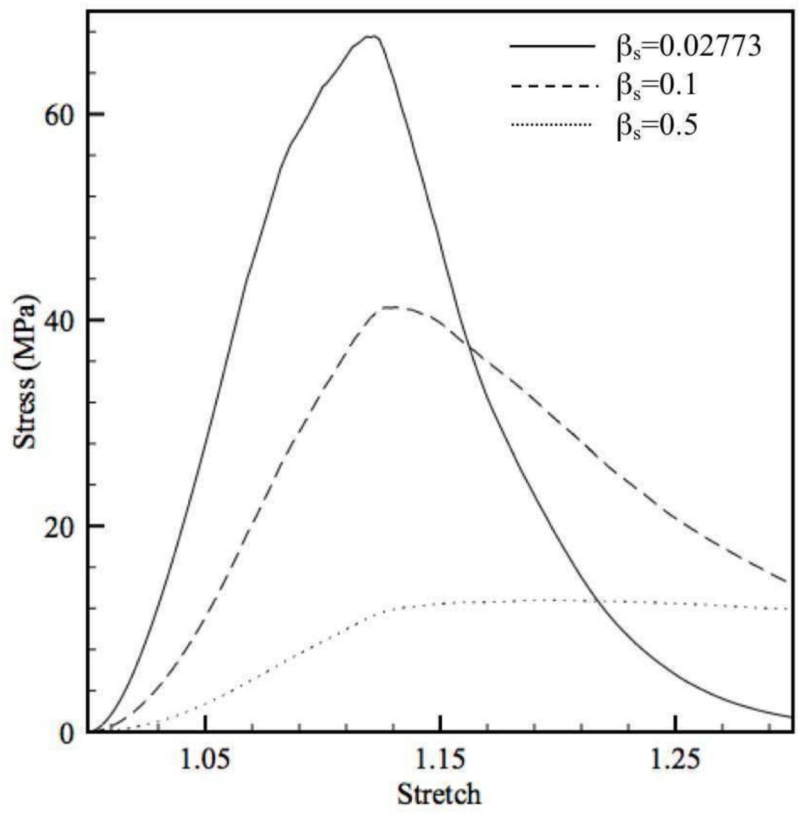

Figure 4. Model predictions of different β_s values

Results: Varying Damage Parameters

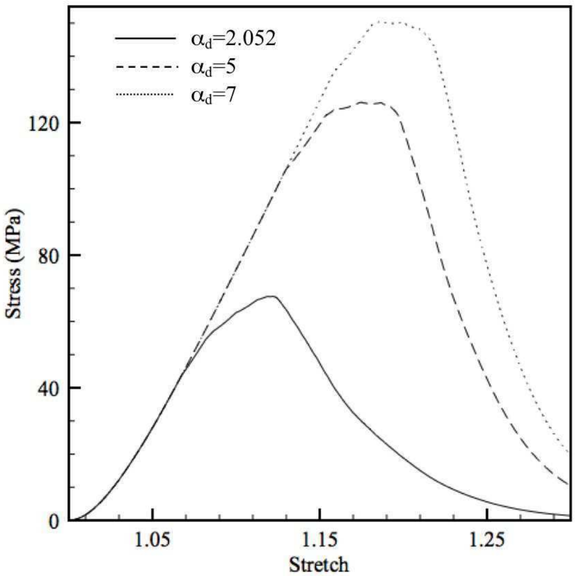

Figure 5. Model predictions of different α_d values

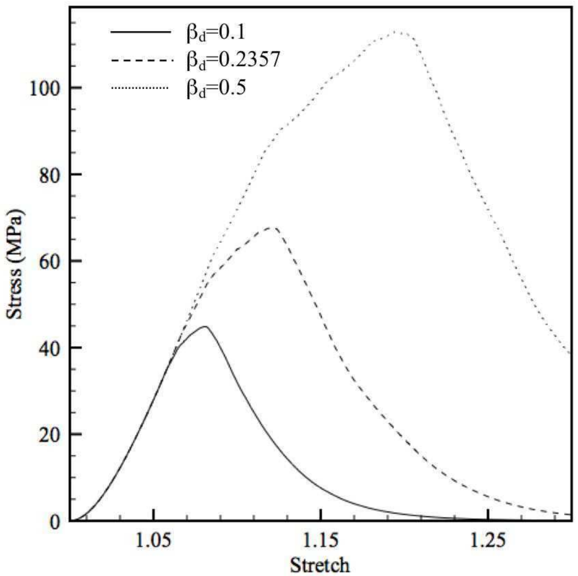

Figure 6. Model predictions of different β_d values

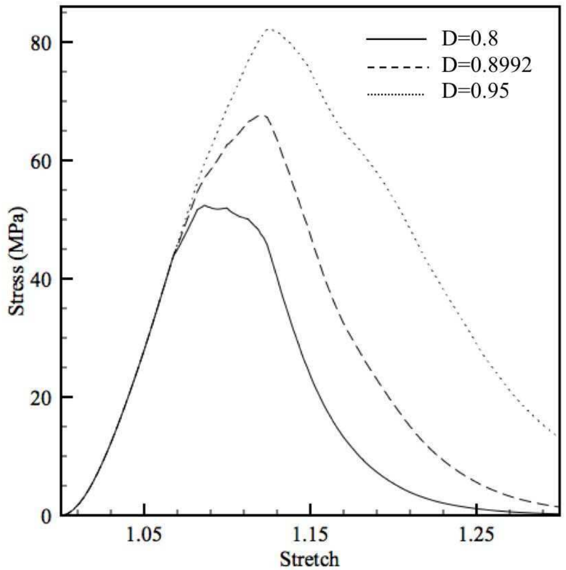

Figure 7. Model predictions of different D values

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Conclusions

- One-dimensional model has been presented to describe the tensile and damage behavior of ligaments

- The model was fitted to the tensile experimental data for rat MCL

- It can be extended to 3-D model by introducing the fiber orientations

Thank you!

Questions