The History of the Second Lunar Anomaly

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http://people.scs.fsu.edu/~dduke/articles.htm

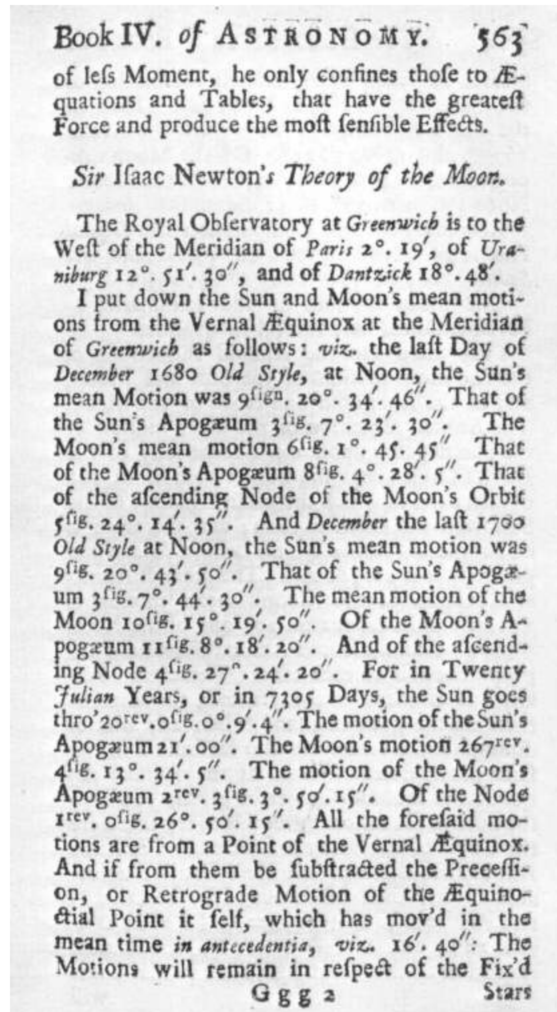
$$q = -6;18^\circ \sin\alpha + 0;13^\circ \sin 2\alpha - 1;16^\circ \sin(2\eta - \alpha) + 0;40^\circ \sin 2\eta + \ldots$$

$\psi = L' - A$ be the elongation of the mean Sun from the lunar apsidal line
$\eta = L - L'$ be the elongation of the mean Moon from the mean Sun
$\alpha = L - A$ be the Moon's mean anomaly, then clearly $\alpha = \eta + \psi$, so note that $2\alpha = 2\eta + 2\psi$

We'll tour backward in time…..

beginning at the end…..

Isaac Newton, *The Motion of the Moon* (1702)



Book IV. *of* ASTRONOMY. 563

of leſs Moment, he only confines thoſe to Æquations and Tables, that have the greateſt Force and produce the moſt ſenſible Effects.

Sir Iſaac Newton's *Theory of the Moon.*

The Royal Obſervatory at *Greenwich* is to the Weſt of the Meridian of *Paris* 2°. 19′, of *Uraniburg* 12°. 51′. 30″, and of *Dantzick* 18°. 48′.

I put down the Sun and Moon's mean motions from the Vernal Æquinox at the Meridian of *Greenwich* as follows: *viz.* the laſt Day of *December* 1680 *Old Style*, at Noon, the Sun's mean Motion was 9$^{\text{ſign}}$. 20°. 34′. 46″. That of the Sun's Apogæum 3$^{\text{ſig}}$. 7°. 23′. 30″. The Moon's mean motion 6$^{\text{ſig}}$. 1°. 45. 45″ That of the Moon's Apogæum 8$^{\text{ſig}}$. 4°. 28′. 5″. That of the aſcending Node of the Moon's Orbit 5$^{\text{ſig}}$. 24°. 14′. 35″. And *December* the laſt 1700 *Old Style* at Noon, the Sun's mean motion was 9$^{\text{ſig}}$. 20°. 43′. 50″. That of the Sun's Apogæum 3$^{\text{ſig}}$. 7°. 44′. 30″. The mean motion of the Moon 10$^{\text{ſig}}$. 15°. 19′. 50″. Of the Moon's Apogæum 11$^{\text{ſig}}$. 8°. 18′. 20″. And of the aſcending Node 4$^{\text{ſig}}$. 27°. 24′. 20″. For in Twenty *Julian* Years, or in 7305 Days, the Sun goes thro' 20$^{\text{rev}}$. 0$^{\text{ſig}}$. 0°. 9′. 4″. The motion of the Sun's Apogæum 21′. 00″. The Moon's motion 267$^{\text{rev}}$. 4$^{\text{ſig}}$. 13°. 34′. 5″. The motion of the Moon's Apogæum 2$^{\text{rev}}$. 3$^{\text{ſig}}$. 3°. 50′. 15″. Of the Node 1$^{\text{rev}}$. 0$^{\text{ſig}}$. 26°. 50′. 15″. All the foreſaid motions are from a Point of the Vernal Æquinox. And if from them be ſubſtracted the Preceſſion, or Retrograde Motion of the Æquinoctial Point it ſelf, which has mov'd in the mean time *in antecedentia*, *viz.* 16′. 40″: The Motions will remain in reſpect of the Fix'd

Ggg 2 Stars

Newton once said that "my head never ached but with the study of the Moon."

Jeremiah Horrocks (1618–1641)

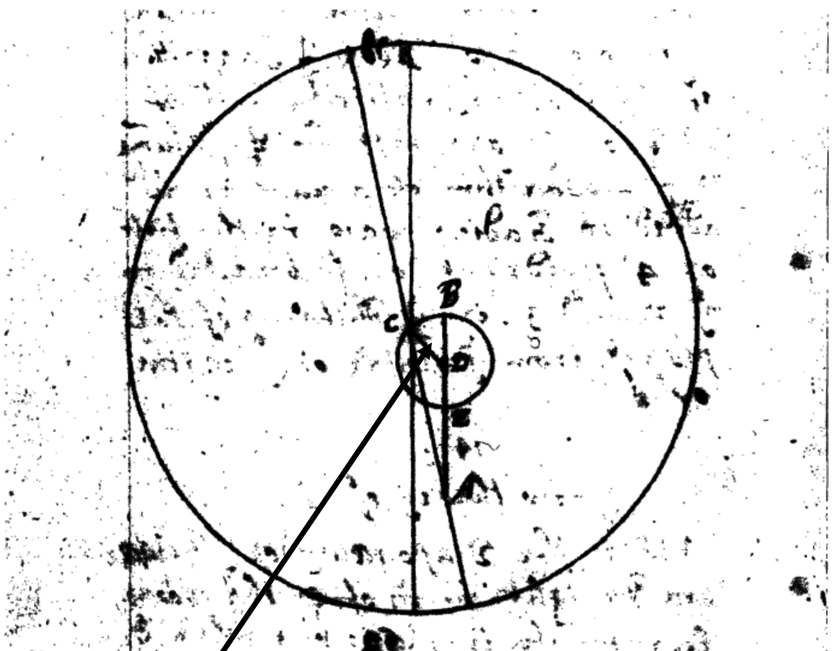


FIG. 4. Horrocks's lunar theory (from his *Philosophicall exercises*, fol. 82v, photograph courtesy of Royal Greenwich Observatory).

2ψ

The earth is at A, the big circle is the Moon's orbit, the apsidal direction and eccentricity AC rotates about the mean position AD with a period of about 6.5 months. Horrocks' theory was purely kinematical, but Newton managed to compute the *periods* from first principles.

Based on an intermediate stage of Kepler’s (1617-18) lunar theory.

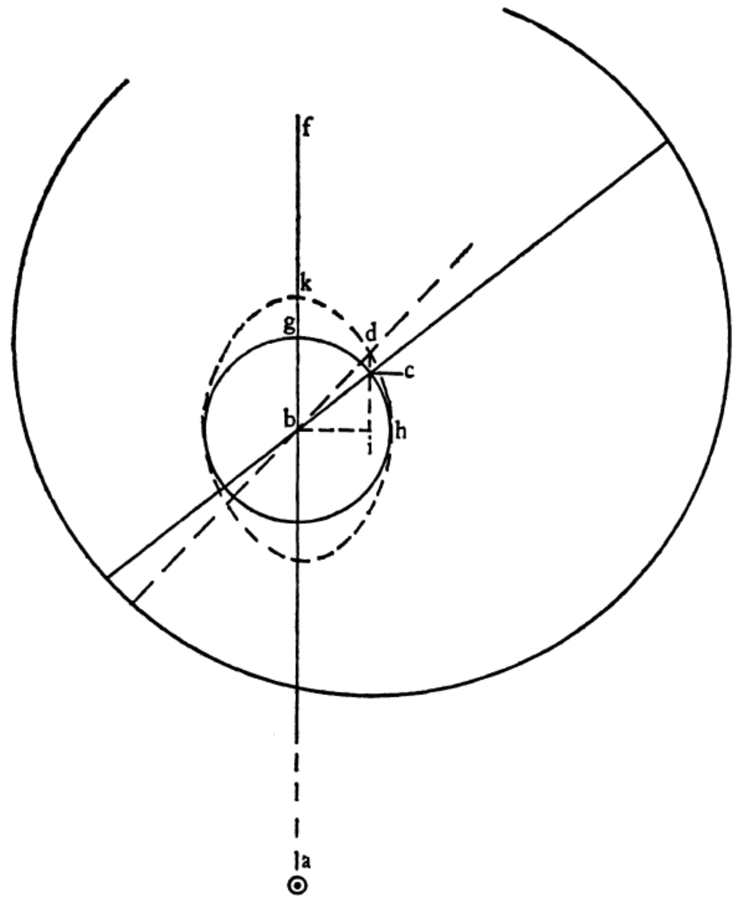


FIG. 1. Kepler’s lunar theory, 1617 and 1618 (from Kepler, *Gesammelte Werke*, xi/1, 522).

Horrcks was also aware of a similar theory of Philippe van Lansberge. However this theory is modeled after Copernicus, so the period of rotation of the point M is about 15 days (semi-monthly)

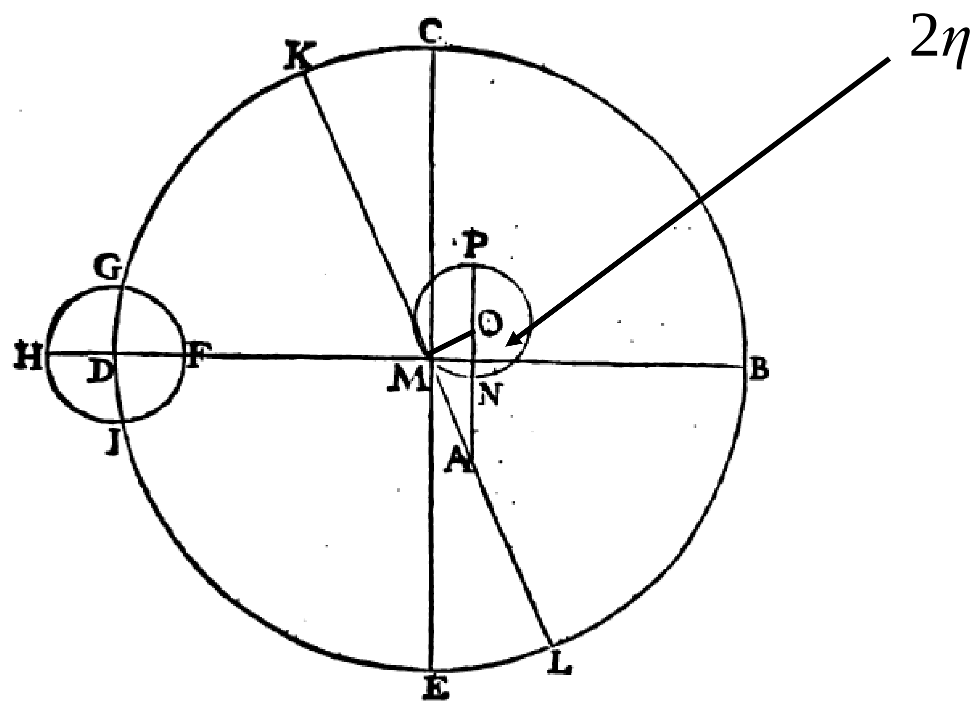


FIG. 3. Lansberge's lunar theory.[14]

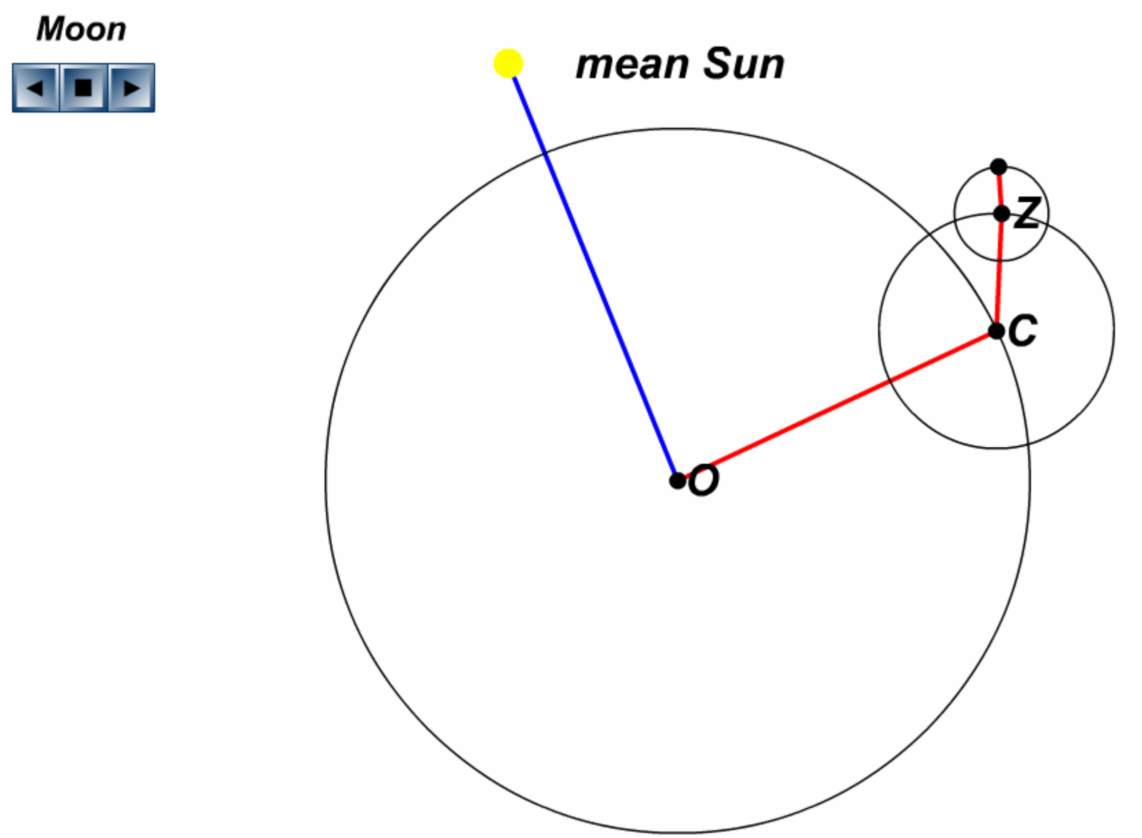
This has led to some confusion by modern historians. For example, C. Wilson, JHA 1987:

the Moon moves uniformly; *K* is the apogee. Lansberge stipulates that *M* rotate about the circle *PMN*, so that the eccentricity varies periodically in value, and the line of apsides oscillates to either side of its mean position. When Horrocks comes to represent his final theory in a diagram, he will use essentially the same figure, except for a change of letters (see Figure 4). The two theories are in fact very different. According to Lansberge, the revolution of *M* in the small circle is completed twice each month, and the eccentricity *AM* is the unbisected eccentricity of pre-Keplerian astronomers. According to Horrocks's final theory, the orbit is an ellipse on which the motion is non-uniform in the way Kepler's areal rule requires, and the period of rotation of the centre of the orbit about the small circle is not half a month but over half a year.

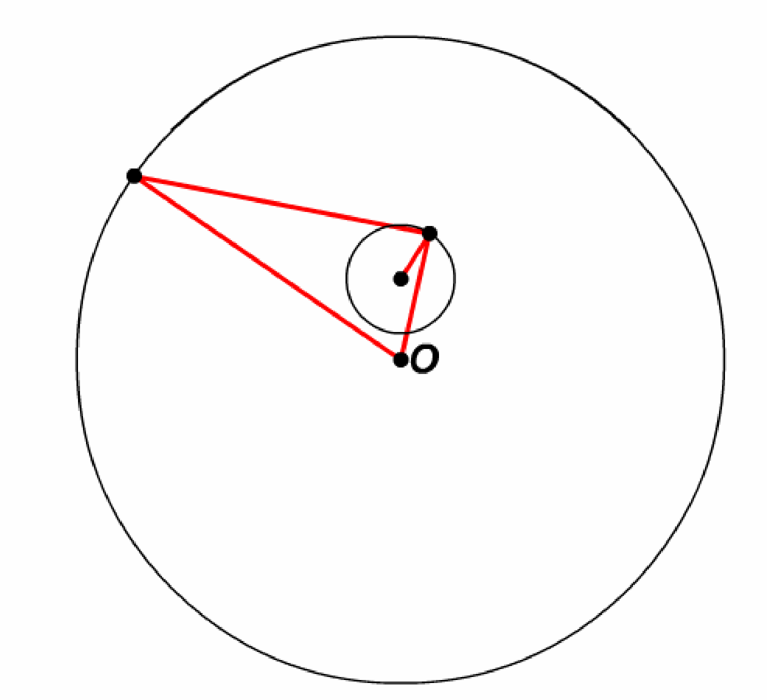
and Nauenberg, JHA 2001:

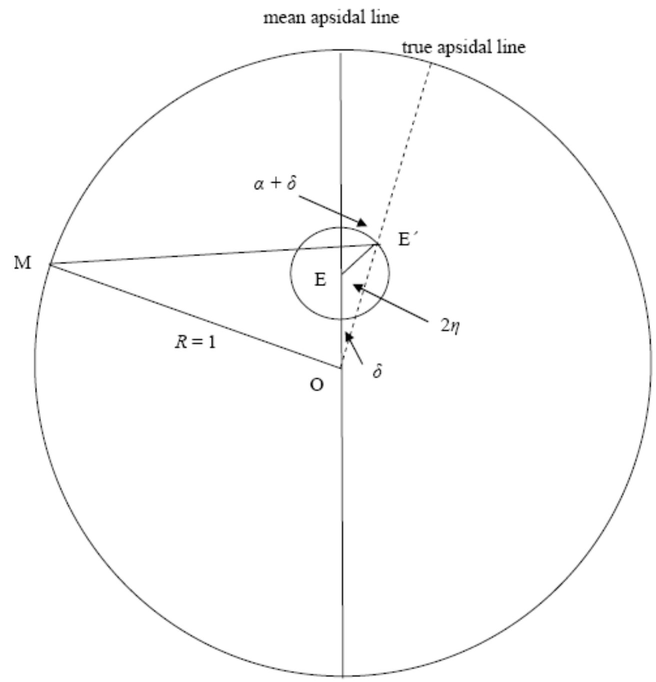
for planetary motion into his lunar model. Finally, in 1640 a young astronomer, Jeremiah Horrocks,[6] refined Kepler's model further, predicting correctly the inequalities in the distance of the Moon from the Earth that were being determined at that time by a micrometer. By setting the centre of the Moon's elliptic orbit on an epicycle, Horrock's model gave rise to an oscillating eccentricity and line of apses. Moreover, for the angle of rotation of this centre, Horrocks adopted Kepler's choice of twice the angle between the Earth–Sun direction and the mean line of apses of the orbit of the Moon. Since Greek times, the corresponding angle chosen to describe Ptolemy's evection inequality was twice the elongation of the Moon from the Sun, but it is the Kepler-Horrocks angle that turns out to be correct as shown by Newton in his dynamical formulation of this inequality.[7]

Now Copernicus, of course, used the model of al-Shatir, and here is how the model works:
http://people.scs.fsu.edu/~dduke/copernic.html
The small epicycle revolves twice-monthly.

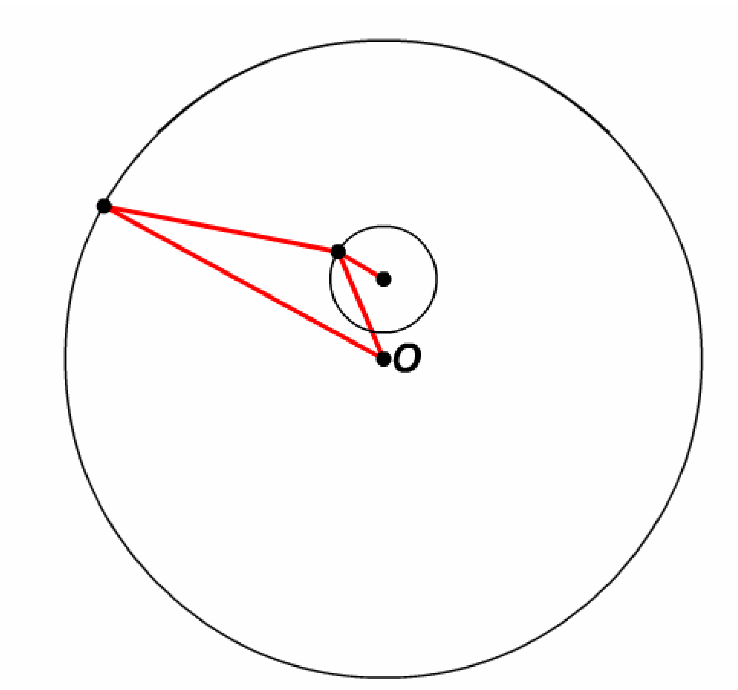


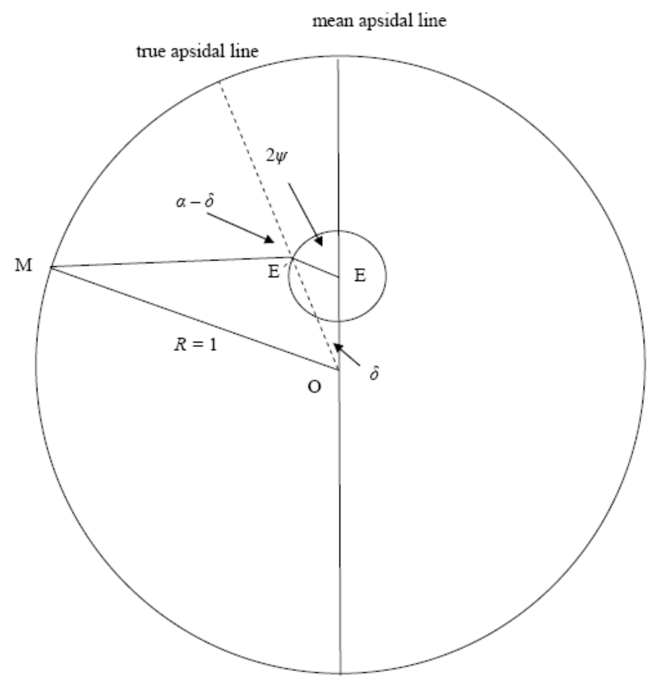
If you move the double epicycle to a concentric construction you get: http://people.scs.fsu.edu/~dduke/shatir.html and this is close to the van Landsberge eccentric model since the small epicycle is still rotating bi-monthly.





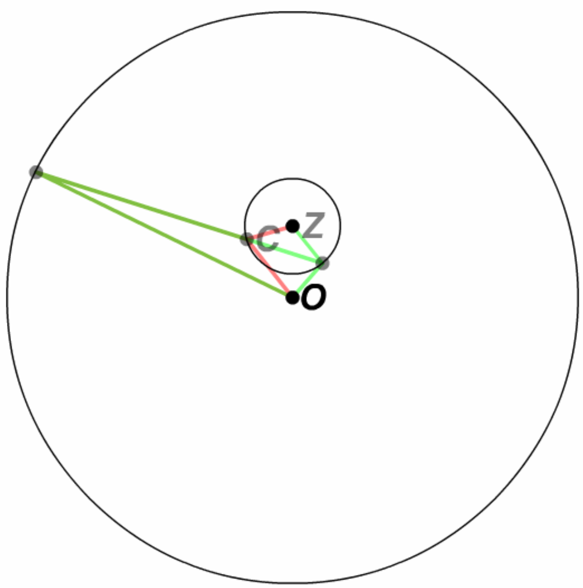
the Kepler/Horrocks/Newton model is very similar, except the small epicycle is rotating approximately *bi–annually*.
http://people.scs.fsu.edu/~dduke/munjala.html

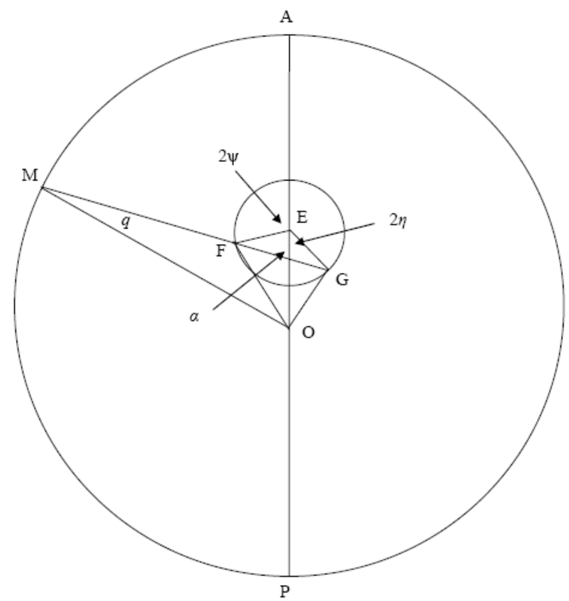




what happens if you put these together?
http://people.scs.fsu.edu/~dduke/munjala-shatir.html

The models are essentially identical:

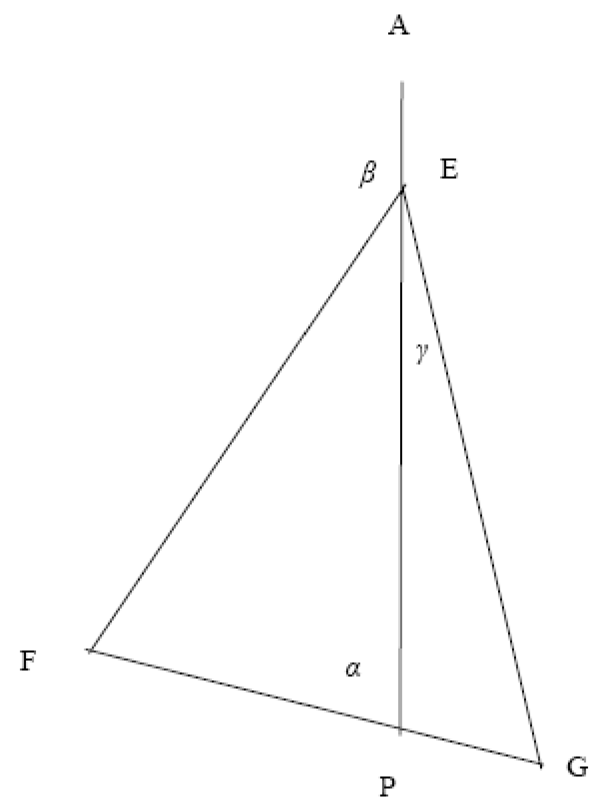




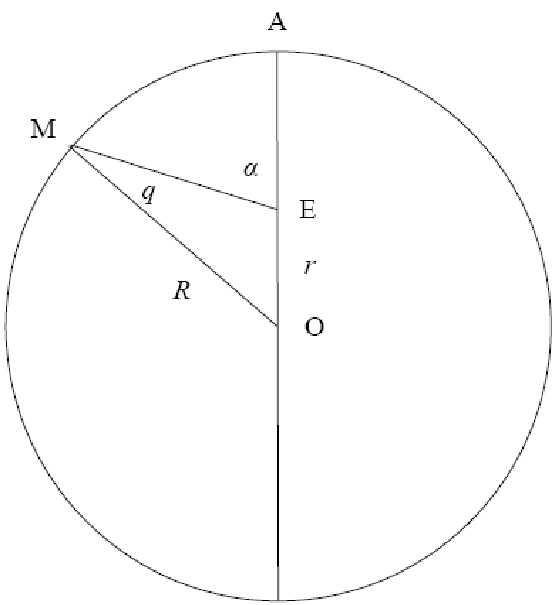
Recall that $2\alpha = 2\eta + 2\psi$ or $2(L-A) = 2(L-L') - 2(L'-A)$
But this also follows from the geometry of the figure.

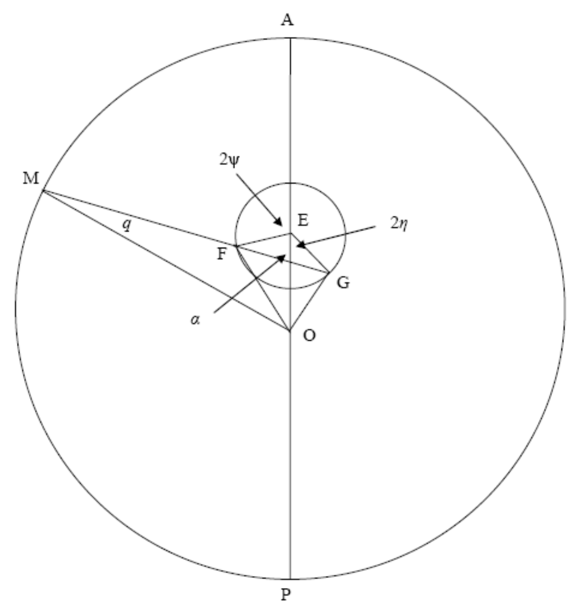
Theorem. Let EFG be an isosceles triangle, with EF = EG. From the apex E drop a line to an arbitrary point P on the base FG, and extend the line above E to point A. Let angle AEF = β, angle PEG = γ, and angle EPF = α. Then $2\alpha = \beta + \gamma$.

Corollary. Let angle β be measured counterclockwise from EA, and let angle γ be measured counterclockwise from EP. Let angles β and γ change uniformly with time so that $\beta = \beta_0 + \omega_\beta t$ and $\gamma = \gamma_0 + \omega_\gamma t$. Then angle EPF = α, measured counterclockwise from PE, also changes uniformly with time, and with speed $2\omega_\alpha = \omega_\beta + \omega_\gamma$

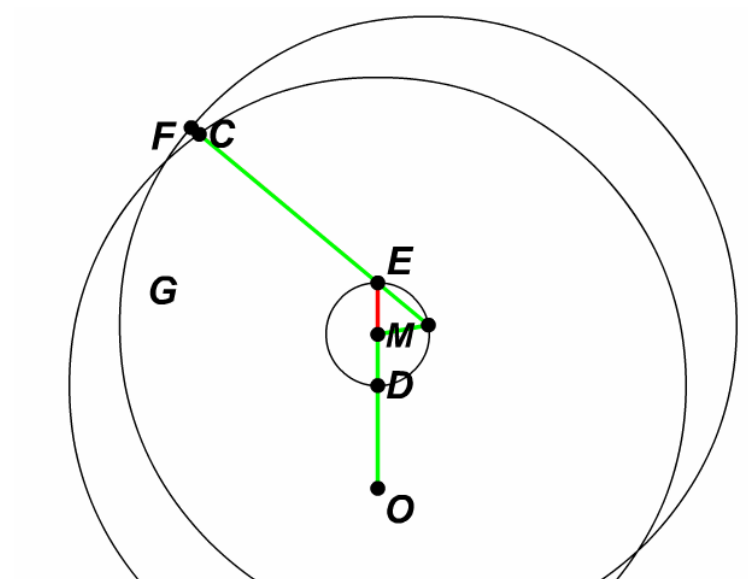


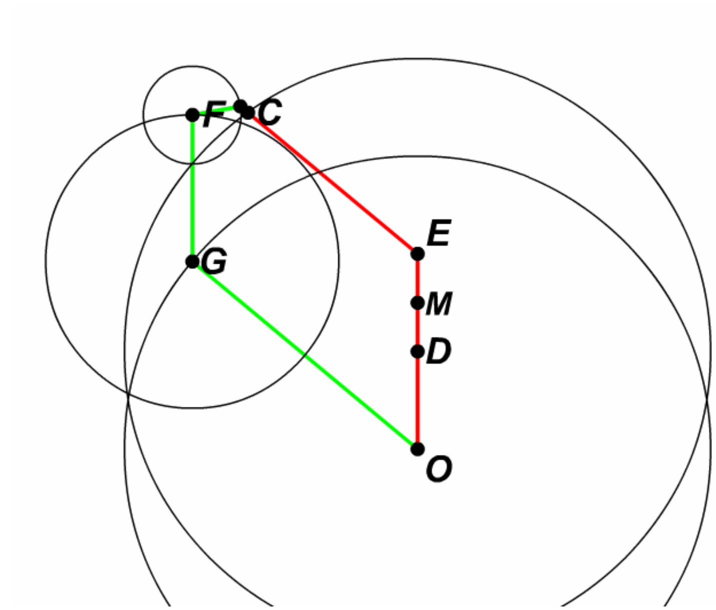
The model has two interesting special cases. First, if we let $\beta = \gamma = \alpha$ and $\omega_\beta = \omega_\gamma = \omega_\alpha$ then the two radii EF and EG will always point in opposite directions and FEG will be a straight line which always makes an angle α with the apsidal line AP. This is exactly the concentric equant used for the first anomaly alone, and we see that one way of understanding *the full lunar model is as a generalization of the concentric equant* to allow $\beta \neq \gamma$ while keeping $\beta + \gamma = 2\alpha$.





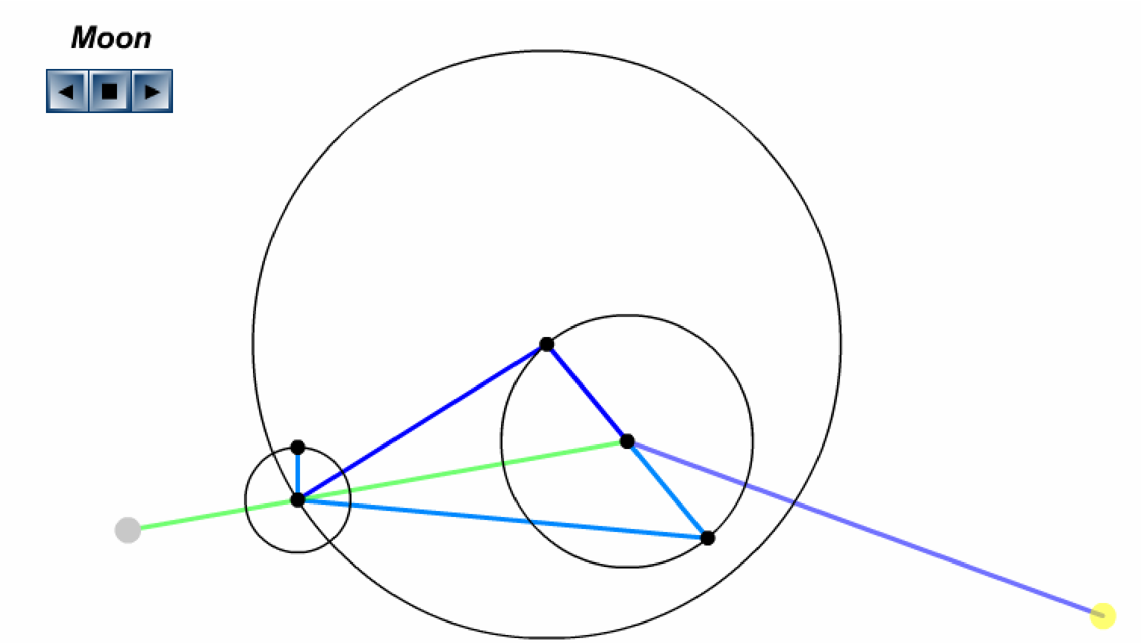
A second special case has $\beta = 0$ and $\omega_\beta = 0$, and $\gamma = 2\alpha$ and $\omega_\gamma = 2\omega_\alpha$. Then the radius EF points toward A and stays fixed, and the radius EG rotates counterclockwise with speed $2\omega_\alpha$. This model is then very closely related to the planetary models of al-Shatir with (a) exchange of the concentric equant with an eccentre or equivalent large epicycle and (b) putting the small epicycle in various places in the figure. One such variant – putting the small epicycle on the tip of the large epicycle – was adopted by Copernicus. http://people.scs.fsu.edu/~dduke/shatir-planets.html





The lunar model in the *Almagest* is in fact also a modified concentric equant, but Ptolemy puts the Earth not at the center but at the equant point itself. and in the apsidal direction, but not the eccentricity, oscillates around a mean value. http://people.scs.fsu.edu/~dduke/moon10a.html

$$\tan q = \frac{-r\sin(2\eta-\delta)}{\rho + r\cos(2\eta-\delta)}.$$



But we have skipped a very interesting chapter in this story:

The first two terms in modern lunar theory are conventionally written as

$$-2e\sin\alpha - \varepsilon\sin(2\eta - \alpha),$$

The two modern terms can be rewritten as

$$\begin{aligned} -2e\sin\alpha - \varepsilon\sin(2\eta - \alpha) &= -2e\sin\alpha + \varepsilon\sin\alpha - 2\varepsilon\cos\psi\sin\eta \\ &= -(2e - \varepsilon)\sin\alpha - 2\varepsilon\cos\psi\sin\eta \\ &= -r\sin\alpha - r'\cos\psi\sin\eta \end{aligned}$$

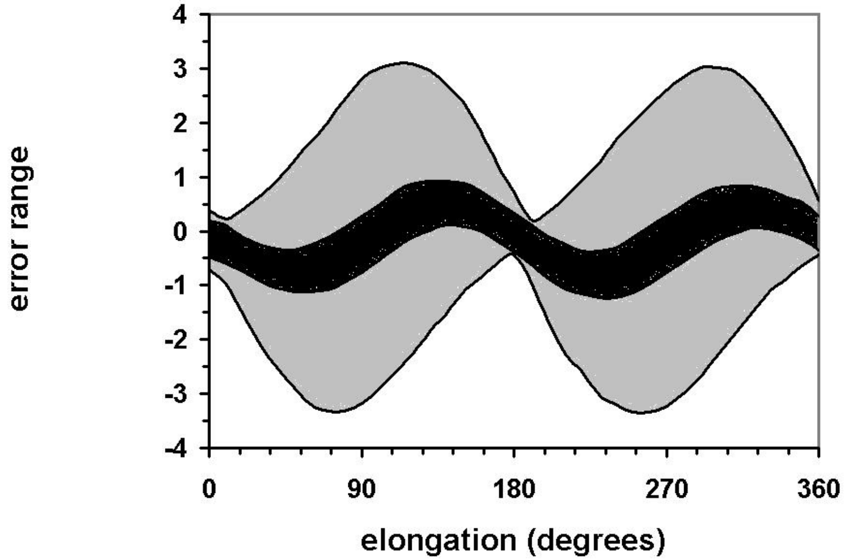
and this last line is found *exactly* in ancient Indian astronomy.

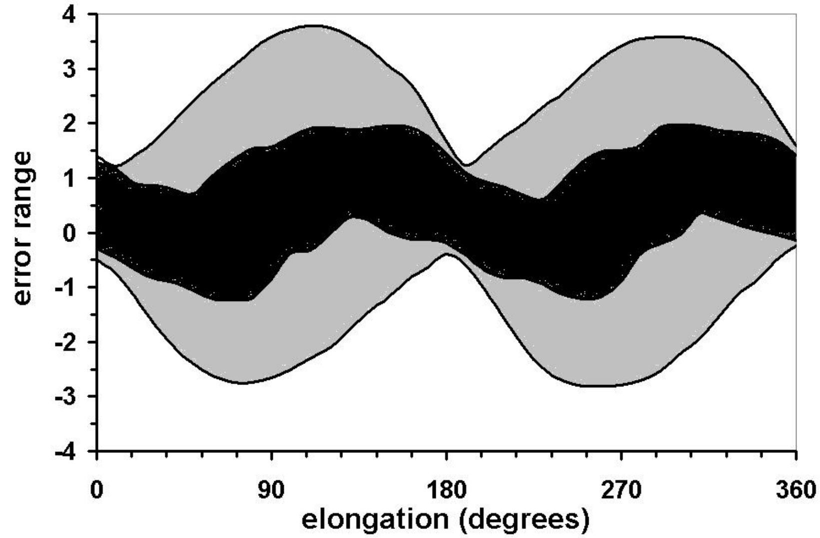
The *Laghumanasam*, a short text by Munjala probably written around A.D. 930, gives much of the standard Indian planetary model information and appears to be derived from Aryabhata's various texts, written *ca.* A.D. 500, Brahmagupta's *Brahmasphutasiddhanta*, *ca.* A.D. 628, and the *Suryasiddhanta, ca.* sixth century A. D. The *Laghumanasam* is a type of text known as a karana, which is a short work giving simplified and approximate rules for computing astronomical items. Among the rules that Munjala gives is a correction to the equation of center for the Moon in the form

$$\sin q = -r \sin\alpha - r' \cos\psi \sin\eta$$

which agrees *exactly* with modern theory. Yallaya, in a commentary to the *Laghumanasam* written in A.D. 1428, claims that this correction was given earlier by Vatesvara (*ca.* A.D. 904), but that earlier text has not been found.

Munjala’s errors (left) are much smaller than the *Almagest*’s (right).

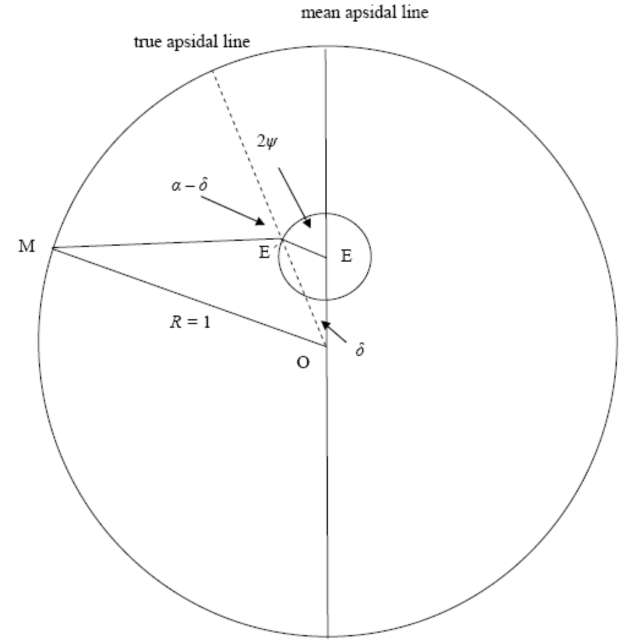




Unlike the modern theoretical expression, in which the evection term $-\sin(2\eta-\alpha)$ does not vanish at syzygy, Munjala’s expression neatly isolates from the evection the part that contributes only away from syzygy. Munjala’s expression is in fact a simple correction to the traditional Hipparchan first inequality.

Consider again this figure.

The equant eccentricity ρ oscillates between $2e-\varepsilon$ and $2e+\varepsilon$, and the true lunar apsidal line oscillates about the mean apsidal direction by an angle δ. Thus at any instant the model is a concentric equant with oscillating eccentricity and apsidal line, so

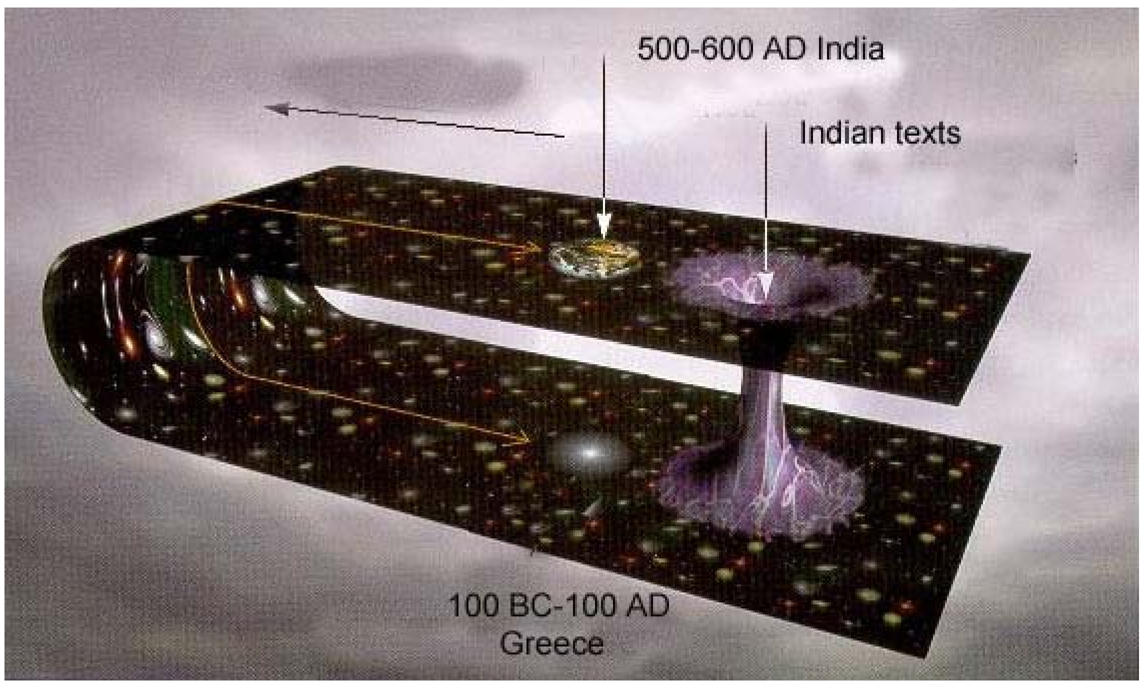


letting
$$\rho\cos\delta = 2e+\varepsilon\cos 2\psi$$
$$\rho\sin\delta = \qquad \varepsilon\sin 2\psi$$

$$\begin{aligned}\sin q &= -\rho\sin(\alpha-\delta)\\ &= -2e\sin\alpha-\varepsilon\sin(\alpha-2\psi)\\ &= -2e\sin\alpha-\varepsilon\sin(2\eta-\alpha)\\ &= -(2e-\varepsilon)\sin\alpha-\varepsilon\cos\psi\sin\eta\\ &= -r\sin\alpha-r'\cos\psi\sin\eta\end{aligned}$$

While it *could* be the case that Munjala's theoretical expression is derived from some Greek geometrical model such as the crank model given in the *Almagest*, it seems more likely that Munjala's expression is in fact an *exact* consequence of a simple underlying geometrical model, which may or may not have even been known to Munjala (or Vatesvara), recalling that under the generally accepted hypothesis upheld by Neugebauer, Pingree, and van der Waerden (among others):

> *The texts of ancient Indian astronomy give us a sort of wormhole through space-time back into an otherwise inaccessible era of Greco-Roman developments in astronomy.*



Summary (reverting at last to historical order):

1. The physical idea of an oscillating lunar eccentricity and apsidal line was apparently known as early as medieval India and perhaps back to Greco-Roman times, if the Neugebauer–Pingree–van der Waerden hypothesis applies.

2. It was used and perhaps re-discovered

- by Arabic astronomers for the Moon and the planets,
- likewise, by Copernicus,
- by various post-Copernican astronomers, including Kepler, van Lansberge, and Horrocks,
- and it was the crucial clue Newton borrowed from Horrocks to finally formulate his own lunar model.

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This talk is based on D. Duke, "The Second Lunar Anomaly in Ancient Indian Astronomy", *Archive for History of Exact Sciences*, 61 (2007) 147–157.
See http://people.scs.fsu.edu/~dduke/2nd_lunar_anomaly.htm for the models

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