http://people.sc.fsu.edu/~jburkardt/presentations/geometry_homework2.pdf

Homework #13

Algorithms for Science Applications II
Assigned: Friday, 22 April 2011
Due: Friday, 29 April 2011

This homework will not be assigned! Instead, you should finish up your projects!

The numerical values used in the six point quadrature rule are available on Blackboard, or in http://people.sc.fsu.edu/~jburkardt/latex/asa_2011_geometry_homework/triangle_quadrature_six.m

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Problem 1: Quadrature Rules for the Unit Triangle

By the *unit triangle* we mean the triangle **T1** whose vertices are A1 = (1,0), B1 = (0,1), C1 = (0,0). A common problem in calculus involves determining the integral of some function $f(x, y)$ over the unit triangle, which can be symbolized formally as:

$$\int_{T1} f(x,y) = \int_0^1 \int_0^{1-x} f(x,y)\,dy\,dx$$

A *quadrature rule* for the triangle **T1** is a method of approximating integrals by supplying a set of N points $(x1_i, y1_i)$ and corresponding weights w_i, where Area(T1) is, of course $\frac{1}{2}$, so that:

$$\int_{T1} f(x,y) \approx \text{Area}(T1) \cdot \sum_{i=1}^{N} w_i\, f(x1_i, y1_i)$$

Here is a quadrature rule for **T1** that uses 3 points:

W	X1	Y1
$\frac{1}{3}$	0	$\frac{1}{2}$
$\frac{1}{3}$	$\frac{1}{2}$	0
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$

and a quadrature rule for **T1** using 6 points:

W	X1	Y1
0.109951743655322	0.816847572980459	0.091576213509771
0.109951743655322	0.091576213509771	0.816847572980459
0.109951743655322	0.091576213509771	0.091576213509771
0.223381589678011	0.108103018168070	0.445948490915965
0.223381589678011	0.445948490915965	0.108103018168070
0.223381589678011	0.445948490915965	0.445948490915965

Using the 3 point and 6 point quadrature rules, estimate the following integrals over the unit triangle:

- $f(x, y) = 1$ (your answer should be $\frac{1}{2}$!);
- $f(x, y) = x^2 + y^2$;
- $f(x, y) = \sin(xy)$.

Turn in: Your two estimates of each integral.

Problem 2: Quadrature Rules for an Arbitrary Triangle

Now suppose that we are interested in approximating the integral of a function over an arbitrary triangle T. If we have a quadrature rule with weights W and points (X1,Y1) for the unit triangle $T1$, then we can adapt it for the triangle T by converting each (X1,Y1) point in $T1$ to an (X,Y) point in T, using the formula:

- X = A1(x) * X1 + B1(x) * Y1 + C1(x) * (1 - X1 - Y1);
- Y = A1(y) * X1 + B1(y) * Y1 + C1(y) * (1 - X1 - Y1);

Here, $A1(x)$ means the x coordinate of the $A1$ vertex of the $T1$ triangle, and so on.

Our resulting quadrature rule is now:

$$\int_T f(x,y) \approx \text{Area}(T) \cdot \sum_{i=1}^{N} w_i \, f(x_i, y_i)$$

Let the triangle T have vertices A=(0,1), B=(3,0), C=(2,5) and:

- Transform the six point rule from the unit triangle $T1$ to our new triangle T;
- Use this rule to estimate the integral of $f(x,y) = x^2$ over T.

Turn in:

- A table of X and Y for the transformed six point rule over T;
- Your estimate for the integral of $f(x,y)$ over T.
