Strategic Formation of Cooperative Networks

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Abstract

Models of public goods in networks typically rely on the realization of a static network structure between participating entities. In many situations, however, for example in ad-hoc and peer to peer information sharing networks, it may not be the case that the underlying network is fixed. Indeed, it may even be the case that nodes are able to select the other nodes to whom they would like their contributions to flow directly, and have the freedom to alter the direction of these outgoing value spillovers over time. We extend a model of contributions in public goods games to incorporate a dynamic and endogenous process of network formation, using a structural approach. By developing reciprocal preferences which are compatible with a potential game formulation, we derive a quantal response equilibrium which provides an expected distribution over not only contributions within a network structure, but over the network structure as a whole. This model is a generalization of a class of mean-field structural models known as Exponential [Family] Random Graph Models or ERGMs. We discuss an experiment designed to test the role of behavioral preferences in strategic network formation, understand the characteristics of equilibrium networks, and observe how participants change their behavior when provided with information about others' characteristics, such as reputation.

Introduction

How and why do individuals share with each other, in situations where it may seem illogical to do so under the standard assumption of rational self-interested preferences? Much prior work has been aimed at uncovering the root causes of this phenomenon, and has lead to a multitude of theories surrounding the consistent deviation of individuals from canonical equilibrium behavior. In this paper, we hope to unify theories of prosociality through the structural estimation of individual preferences in a novel controlled laboratory setting.

While the decision environment which we study will share some similarities with dictator games, standard linear public goods games, networked public goods games, and club goods, the introduction of a mechanism for endogenous network formation makes this environment truly unique – both in substance and through its ability to statistically isolate and distinguish between different forms of reciprocity and prosociality.

Further, this paper contributes to the new and growing literature on the formation of (pro-)social networks; a notoriously complex problem to analyze and one with far reaching consequences and applications. By limiting the size of the network, and collecting data on dynamic behavior, we aim to gain some insight into the question of how social networks form and evolve to stability.

Objectives

1. Understand network formation in a voluntary contributions setting with a congestible resource
2. Develop a novel structural model of behavioral preferences in resource sharing environments
3. Examine the impact of providing various reputation information on the structure of networks

Theoretical Model

We can represent player decisions in a single period by a weighted, directed networks with implicit self-links. Consider a set of players $N = \{1 \dots n\}$. A player in this set is denoted by i. Players begin each round with an initial endowment ω, and complete a work task to increase their beginning balance by y_i. Each player is asked to simultaneously choose values for c_i, their contribution to the group exchange, and N_i, the set of other individuals to be included in the allocation of their augmented contributions. The vector of all players contributions is denoted by $C = [c_1 \dots c_n]$. Each player's network N_i must always contain at least themselves, $\forall i \in N, i \in N_i$. Once players make their decisions, all contributions are multiplied by a factor m_i, and distributed evenly across the players' chosen networks. We define the value of the multiplicative factor m_i to be the derivative of a per-capita return function $R_i : \mathbb{R} \times (\mathbb{Z}_{n+1} \backslash \{0\}) \to \mathbb{R}$, so that $R_i(c_i, |N_i|) = m_i(|N_i|) * c_i$. The definition of a public good implies that $m_i \leq 1$, since otherwise a player has direct monetary incentive to provide the good. Defining m_i as such provides us the freedom to manipulate the marginal per-capita return (MPCR) of an outgoing network N_i in terms of the number of links made by player i, in order to account for asymmetries which may arise as part of the underlying structure of the mechanism.

We define the overall network by the $n \times n$ adjacency matrix G. Elements of G are denoted by G_{ij} where $G_{ij} = 1$ if Player j is included in the benefits of Player i's public good, and $G_{ij} = 0$ otherwise. Since $\forall i \in N, i \in N_i$, the diagonal elements of G are always equal to 1, that is, $\forall i \in N, G_{ii} = 1$. If $N_i = \{i\}$, the payoff to Player i is equal to their initial endowment, plus the additional balance provided to them through their work, and any benefits they receive from inclusion in other players' networks. If their network size is larger than 1, their payoffs will be reduced by their contribution, and will include additional benefit from their portion of their own public good provision. Then players' material payoffs may be obtained from the following formula:

$$\pi_i(G, C) = (\omega - c_i) + \sum_{j=1}^{n} G_{ji} * m_j * c_j \tag{1}$$

Subjects are likely to receive additional utility from some prosocial preferences. For example, under *reciprocity*, subjects gain some additional positive utility by sharing a portion of their public good provision with another player who is sharing some contribution with them. A player's reciprocal concerns should thus depend on both the players' incoming and outgoing links, as well as the overall amount received by both players as a result of a link. We use $\beta_i(G, C)$ to represent the function of a player's behavioral utility. It is important to note that the exclusive use of behavioral concerns which rely on the existence of mutual connections between players is sufficient to reduce the potential equilibria to networks in which players form only mutual links in equilibrium. This reduces the size of the space of possible network equilibria, and substantially reduces the computational load of finding equilibrium networks. Given a group's behavioral considerations, and incorporating bounded rationality through a stochastic preference shock ϵ_i, the overall utility of Player i will then be given by:

$$U_i(G, C; \theta) = \pi_i(G, C) + \beta_i(G, C, \theta) + \epsilon_i \tag{2}$$

These individual preferences can be aggregated as a potential game [6, 1], with the following potential function:

$$Q(G, C; \theta) = \sum_{i=1}^{n} (m_i(G) - 1) * c_i + \sum_{i=1}^{n} \sum_{j>i}^{n} G_{ij} G_{ji} r_{ij}(G, C) \tag{3}$$

If we assume that the random shocks to preferences follow an extreme value type-I distribution, and that the reciprocity function is linear in parameters, then the network will evolve as a Markov chain with a unique stationary distribution over the space of networks, of the form:

$$f(G, C) = \frac{exp[\theta' t(G, C)]}{\sum_{\Omega \in \mathbf{G}} \int_{\mathcal{C}^n} exp[\theta' t(\Omega, \gamma)] d\gamma} \tag{4}$$

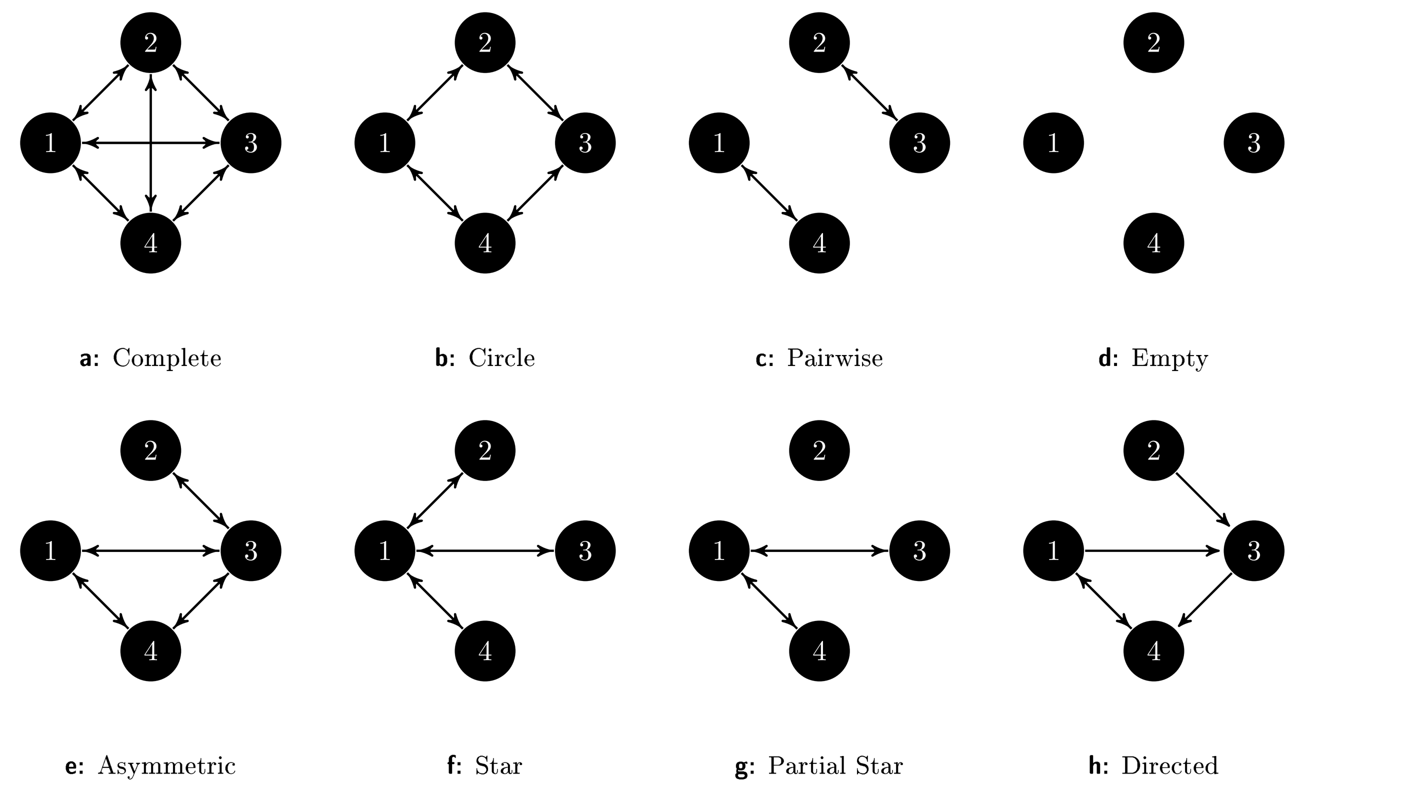

Figure 1: Classification of Network Topologies.
(a): In the case of a *dense* or *complete* topology, the network is identical to a standard public goods game with $n = 4$. (b): A *circle* network is an example of a symmetric, regular graph. Here, any contribution a player chooses is split between themselves and their two neighbors. (c): In the case of a *pairwise matching*, each player chooses one other player to link with, and every link is reciprocated. (d): The *empty* network is equivalent to any network with no contributions; this is the standard Nash prediction. (e): If different players have different numbers of outgoing links or *out degrees*, then the graph is *asymmetric* and they may be facing different MPCRs. (f): A *star* network exhibits a *core-periphery* architecture which is commonly observed in both empirical and experimental studies of network formation. (g): Similar to the star network, but here there is an isolated node, making it a *partial star*. Note that nodes who choose not to link cannot make positive contributions. (h): Remember that here, we are placing no restrictions on who someone can link with; links *do not* need to be a mutual decision. That is, the adjacency matrix G need not be symmetric. A network which exhibits these imbalances in linking is called *directed*.

This is an Exponential family Random Graph Model (ERGM). Which can be fit using maximum likelihood methods, due to the small size of networks. By collecting experimental data on small fixed groups of size four, we can avoid the intractability associated with computation of the normalizing constant for parameter inference in large network ERGM models.

Experiment Design

We hypothesize that providing information which enables direct reciprocity should increase gains from the networks, by making efficient cooperative equilibria more salient. We place subjects in groups of four, and have them play a network sharing game for 15 periods, after which they are told there will be a second part to the experiment, but not told what that will be.

After 15 rounds, subjects are told the rules for the second part of the experiment. In the second part, subjects remain in the same groups but are assigned new identifiers and play another 15 rounds of the game. In treatment sessions, subjects are also shown their incoming benefits from the sharing of others in their group. We observe the effects of this intervention on several outcome variables.

Results

We have conducted some analysis to analyse the impact of providing reputation information on some properties of the networks that form. These include reciprocity, computed using a linear formula $r_{ij}(G, C) = m_i c_i m_j c_j$. Triggering direct reciprocity through the provision of reputation information led to an increase in efficiency gains from the network of more than 100% relative to the baseline treatment.

Table 1: Estimated Treatment Effect of Information Provision (Difference-in-Differences)
This table of regression results highlights the substantial impact of providing reputation information on the structure of the networks that form between experimental subjects.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Efficiency		Reciprocity		Costs		Centralization	
Information Treatment	0.0930* (0.0225)	0.166* (0.0281)	61.17* (12.37)	70.81* (13.69)	0.775* (0.165)	1.214* (0.227)	-0.0420† (0.0170)	-0.0746* (0.0281)
Constant	–	0.151* (0.0105)	–	40.29* (6.399)	–	2.395* (0.181)	–	0.407* (0.0210)
Fixed Effects (Group)	Yes	–	Yes	–	Yes	–	Yes	–
N	1020	1020	1020	1020	1020	1020	1009	1009

Standard errors in parentheses, clustered for 34 groups
* $p < 0.01$, † $p < 0.05$

References

[1] Anton Badev. Nash equilibria on (un) stable networks. 2018.

[2] Andrea Galeotti, Sanjeev Goyal, Matthew O Jackson, Fernando Vega-Redondo, and Leeat Yariv. Network games. *The Review of Economic Studies*, 77(1):218–244, 2010.

[3] Matthew O Jackson and Asher Wolinsky. A strategic model of social and economic networks. *Journal of Economic Theory*, 71(1):44–74, 1996.

[4] Markus Kinateder and Luca Paolo Merlino. Public goods in endogenous networks. *American Economic Journal: Microeconomics*, 9(3):187–212, 2017.

[5] Angelo Mele. A structural model of dense network formation. *Econometrica*, 85(3):825–850, 2017.

[6] Dov Monderer and Lloyd S Shapley. Potential games. *Games and Economic Behavior*, 14(1):124–143, 1996.