CENTROIDAL VORONOI TESSELLATION BASED ALGORITHM FOR VECTOR FIELDS VISUALIZATION AND SEGMENTATION

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Abstract This research extends an existing method for the simplification and the visualization of vector fields based on the notion of Centroidal Voronoi Tessellations (CVT) to time dependent problems. A CVT is a special Voronoi tessellation for which the generators of the Voronoi regions in the tessellation are also the centers of mass (or means) with respect to a prescribed density. A distance function in both the spatial and vector spaces is introduced to measure the similarity of the spatially distributed vector fields. Based on such a distance, vector fields are naturally clustered and their simplified representations are obtained. The method combines simple geometric intuition with the rigorously established optimality properties of the CVT. The algorithm, originally designed for static problems, will be implemented to apply to continuous problems. It will be developed and tested using solutions to the Burgers equation.

Introduction

Large and complex data sets are being generated at an enormously fast speed with the advent of modern computing technology. Effective strategies for data mining that include the representation, simplification, characterization and manipulation of data become increasingly important.

It has always been a computational challenge to visualize large sets of vector fields including those collected from various scientific and engineering disciplines. Here, we propose a clustering/segmentation method for the vector fields based on the notion of Centroidal Voronoi tessellations (CVTs) [Du et al. 1999]. CVTs are optimal tessellations of a given domain and they also give rise to a global approach to cluster a domain into Voronoi regions.

Roughly speaking, for the spatially distributed vector fields of interests to us here, they can be thought as some vector bundles (or fibers) defined in a spatial domain. However, it is more natural and more convenient to treat such vector bundles and the spatial domain together as elements of a higher dimensional manifold equipped with a suitably defined distance (metric). Then, one may obtain, from the higher dimensional distance, a centroidal Voronoi tessellation that defines the clusters of the spatial domain. Then a lifting operation can be applied to obtain the vector representations of the vector fields distributed in each spatial clusters.

Centroidal Voronoi Tessellations

Given an open set $\Omega \subseteq R^N$, the set $\{V_i\}_{i=1}^k$ is called a tessellation of Ω if $V_i \cap V_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^k \overline{V_i} = \overline{\Omega}$ where $\overline{V_i}$ and $\overline{\Omega}$ denote the closures of V_i and Ω. Let d denote a distance defined on R^N. Given points $\{z_i\}_{i=1}^k$ belonging to $\overline{\Omega}$, the Voronoi region (or cluster) $\widehat{V}_i$ corresponding to the point z_i is defined by

$$\widehat{V}_i = \{x \in \Omega | d(x, z_i) < d(x, z_j) \text{ for } j = 1, ..., k, j \neq i\}. \tag{1}$$

The points $\{z_i\}_{i=1}^k$ are called generators. The set $\{\widehat{V}_i\}_{i=1}^k$ is a Voronoi tessellation or Voronoi diagram of Ω, and each $\widehat{V}_i$ is referred to as the Voronoi region corresponding to z_i.

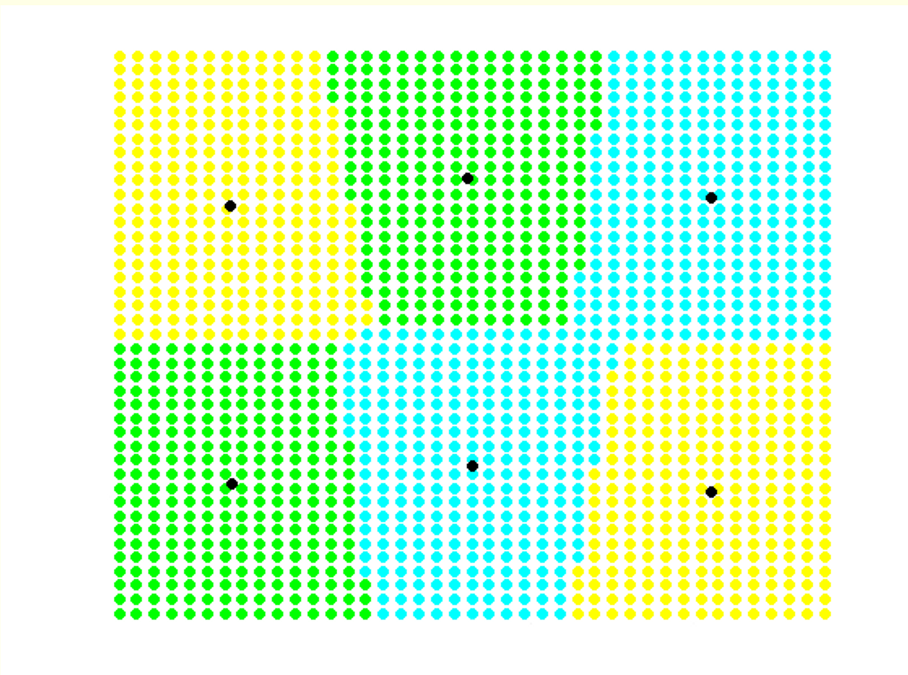

Figure 1: Example of a rectangular domain split into six Voronoi clusters. The generators for each cluster are shown in black.

Vector fields clustering

Given a positive scaling constant w, define the (one-sided) distance between $p = (x_p, y_p)$ and $m = (x_m, y_m)$ as

$$d_p(p, m) = \sqrt{|y_p|^2 - |y_p| y_p \cdot y_m + w|y_p|^2 |x_p - x_m|^2}. \tag{2}$$

Then, given a set of k generators $\{m_i\}_{i=1}^k$ under the constraint $|y_{m_i}| = 1$, the Voronoi regions $\{\widehat{C}_i\}$ corresponding to the point $\{m_i\}$ are defined by

$$\widehat{C}_i = \{x_p \in \Omega | d_p(p, m_i) < d_p(p, m_j) \text{ for } j = 1, ..., k, j \neq i\}. \tag{3}$$

It is obvious that $\widehat{C}_i \cap \widehat{C}_j = \emptyset$ if $i \neq j$. For some p that satisfies $d_p(p, m_i) = d_p(p, m_j)$ for two distinct generators $m_i \neq m_j$, we then assign p to the Voronoi region $\widehat{C}_i$ if $|x_p - x_{m_i}| < |x_p - x_{m_j}|$.

Using the definition of d_p and given a cluster C, the centroid $m*$ is obtained as the minimizer of the energy

$$E(m, C) = \int_C |y_p|^2 - |y_p| y_p \cdot y_m + w|y_p|^2 |x_p - x_m|^2 dx_p. \tag{4}$$

Examples

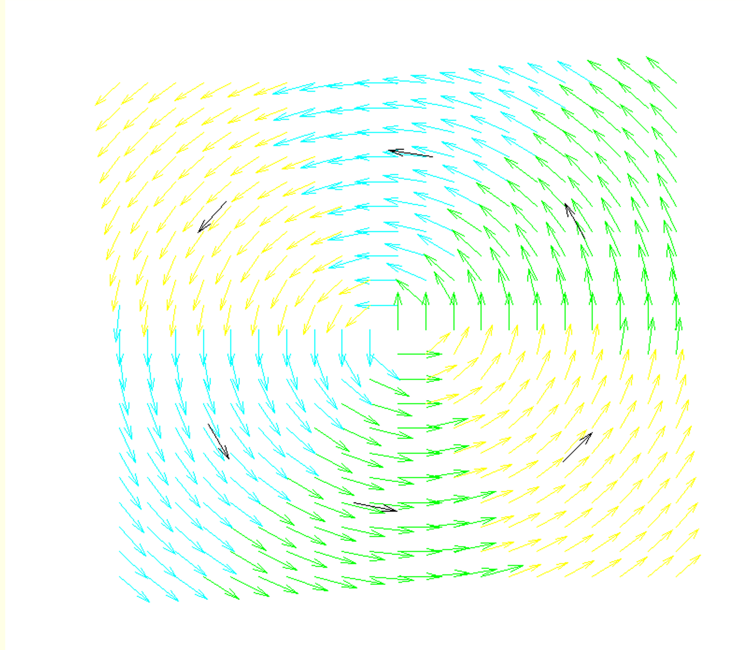

Figure 2: A vector field that was separated into six Vonoroi regions.

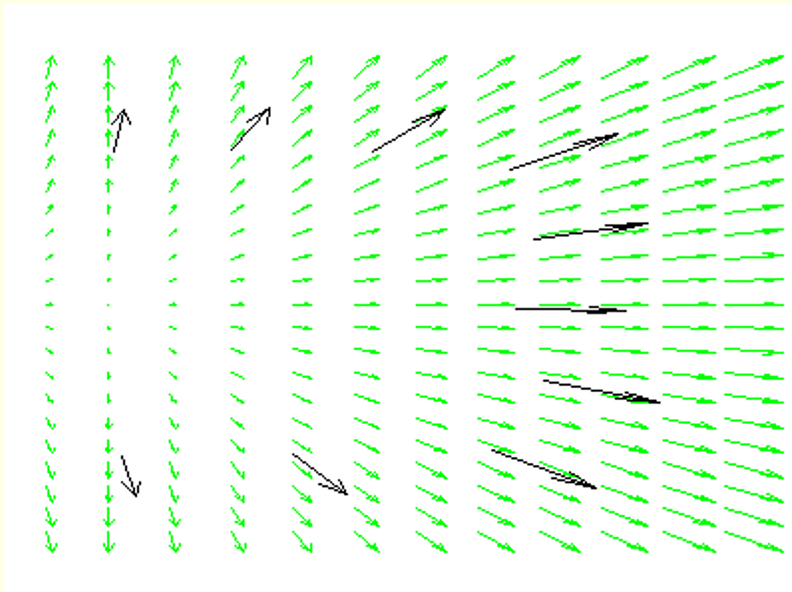

Figure 3: A vector field with vectors of different length separated into ten Vonoroi regions.

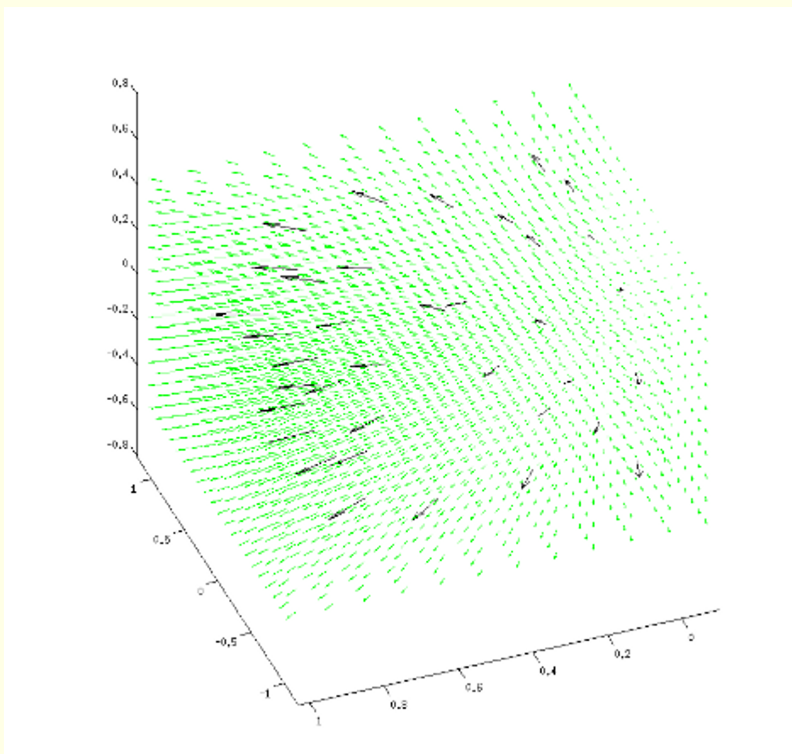

Figure 4: A 3D vector field with vectors of different length separated into 36 Vonoroi regions.

Future Work

At the current time the method has only been used on static problems. The algorithm will be extended to apply to time dependent problems. The new algorithm will use CVT for the visualization of a fluid flow.

Finite element solutions of the Burgers' equation (5) over different domains will be used for implementation and testing purposes. The Burgers' equation is

$$\frac{\partial u}{\partial t} + v_1 u \frac{\partial u}{\partial x} + v_2 u \frac{\partial u}{\partial y} - \frac{\partial}{\partial x}\left(\mu(x,y)\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(\mu(x,y)\frac{\partial u}{\partial y}\right) = f(x,y), \tag{5}$$

where $u = u(t, x, y)$ is a function of time and space, v_1 and v_2 are the convection coefficients, $\mu(x, y)$ is the diffusivity field, and $f(x, y)$ is the forcing term.

References

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