Mesh Sensitivity in Peridynamic Simulations

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1. Introduction

Peridynamics [1, 2] is a recently developed non-local theory of continuum mechanics that is useful in simulating multi-scale phenomena. Its formulation is based upon an integral equation of motion, so that discontinuities may spontaneously form and propagate without special treatment. Thus it is well-suited to modeling materials phenomena that involve discontinuities, such as fracture, dislocations, and phase transitions. In this work, we investigate the suitability of several meshing strategies for simulating a peridynamic brittle impact model. We present a qualitative comparison of the fracture patterns that result, and suggest best practices for generating meshes that lead to efficient, high-quality numerical simulations of peridynamic models.

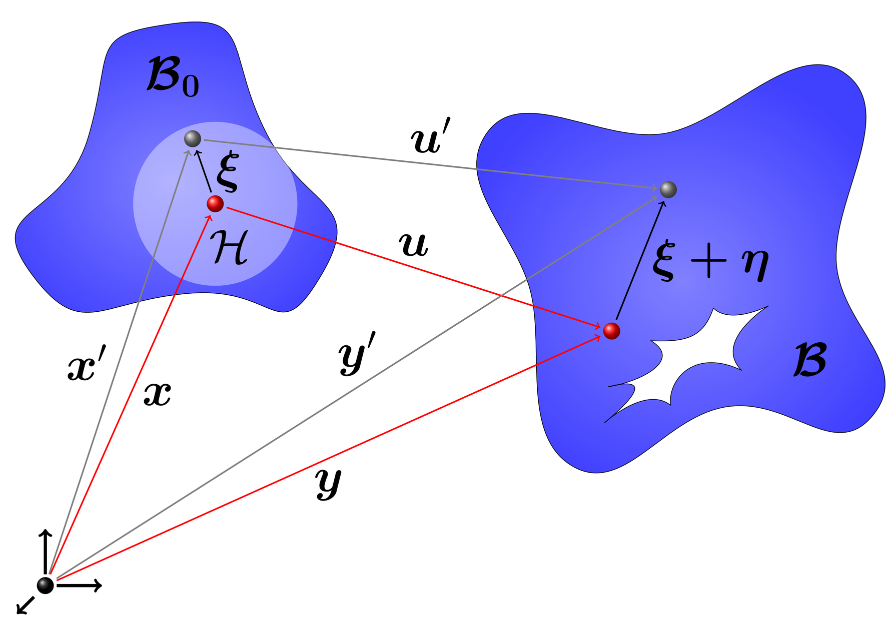


Figure 1: *Kinematic quantities that describe a peridynamic continuum body in its reference ($\mathcal{B}_0$) and current ($\mathcal{B}$) configurations.*

2. Theory

Peridynamic theory is a reformulation of continuum mechanics that employs a non-local force model to account for long-range material interactions. It is governed by an integro-differential equation of motion that avoids spatial derivatives,

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2}(\boldsymbol{x}, t) = \int_{\mathcal{H}_{\boldsymbol{x}}} \boldsymbol{f}(\boldsymbol{u}' - \boldsymbol{u}, \boldsymbol{x}' - \boldsymbol{x})\, dV_{\boldsymbol{x}'} + \boldsymbol{b}(\boldsymbol{x}, t). \quad (1)$$

The pairwise internal force function $\boldsymbol{f}(\boldsymbol{\eta}, \boldsymbol{\xi})$ contains all of a material body's constitutive information and the force term $\boldsymbol{b}(\boldsymbol{x}, t)$ accounts for all external forces acting upon the body.

We restrict our study to micro-elastic materials, in which the pairwise force function is conservative, so $\boldsymbol{f}(\boldsymbol{\eta}, \boldsymbol{\xi})$ can be written as the gradient of a scalar micro-potential,

$$\boldsymbol{f}(\boldsymbol{\eta}, \boldsymbol{\xi}) = \frac{\partial w}{\partial \boldsymbol{\eta}}(\boldsymbol{\eta}, \boldsymbol{\xi}). \quad (2)$$

We postulate that the micropotential can be separated into bonded and non-bonded contributions, so that $w = w^b + w^s$. Bonded particles exert a force on each other that is analogous to an elastic spring,

$$w^b = \frac{1}{2} \frac{c^b}{\|\boldsymbol{\xi}\|} \mu \left(\|\boldsymbol{\eta} + \boldsymbol{\xi}\| - \|\boldsymbol{\xi}\|\right)^2. \quad (3)$$

This expression contains the scalar quantity μ, which tracks the history of damage to each bond. We use a brittle damage model, so that bonds stretched beyond a certain critical extension are broken irreversibly,

$$\mu(t, \xi) = \begin{cases} 1 & s(t', \xi) < s_0(t')\ \forall\, t' \in (0, t) \\ 0 & \text{otherwise} \end{cases}. \quad (4)$$

In addition to the bonded forces, a short-range repulsive force is introduced to prevent the overlap of moving material.

$$w^s = \frac{1}{2} \frac{c^s}{\delta} \left(\|\boldsymbol{\eta} + \boldsymbol{\xi}\| - d^s\right)^2, \quad (5)$$

where d^s is a chosen short-range interaction distance.

3. Numerical Method

Various numerical integration techniques have been useful in approximating the peridynamic equation of motion, including Gaussian quadrature, finite elements, and spectral methods. Our solution scheme uses the so-called mesh-free "EMU" method [3] which discretizes spatial quantities using the composite quadrature rule,

$$\rho \frac{\partial^2 \boldsymbol{u}_i^n}{\partial t^2} = \sum_p \boldsymbol{f}\left(\boldsymbol{u}_p^n - \boldsymbol{u}_i^n, \boldsymbol{x}_p - \boldsymbol{x}_i\right) V_p + \boldsymbol{b}_i^n, \quad (6)$$

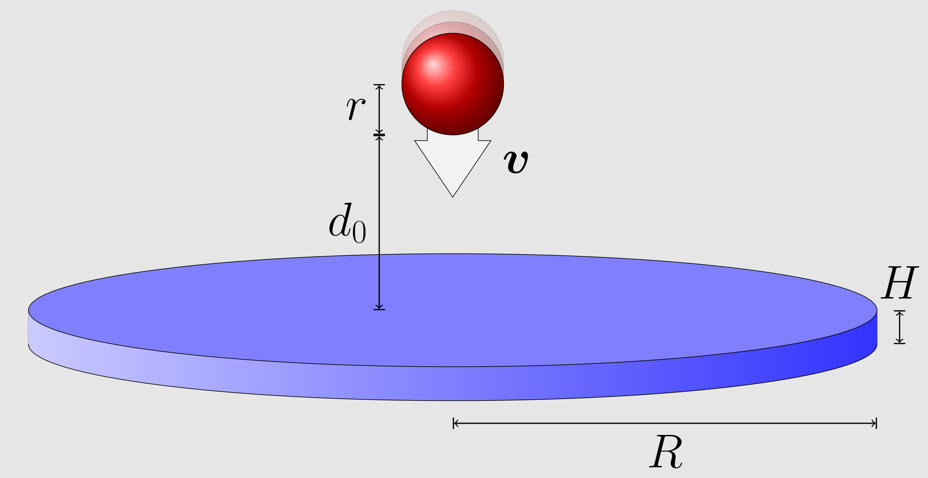
and temporal quantities using a central difference (Verlet) method,

$$\frac{\partial^2 \boldsymbol{u}_i^n}{\partial t^2} \approx \frac{\boldsymbol{u}_i^{n+1} - 2\boldsymbol{u}_i^n + \boldsymbol{u}_i^{n-1}}{(\Delta t)^2}, \quad (7)$$

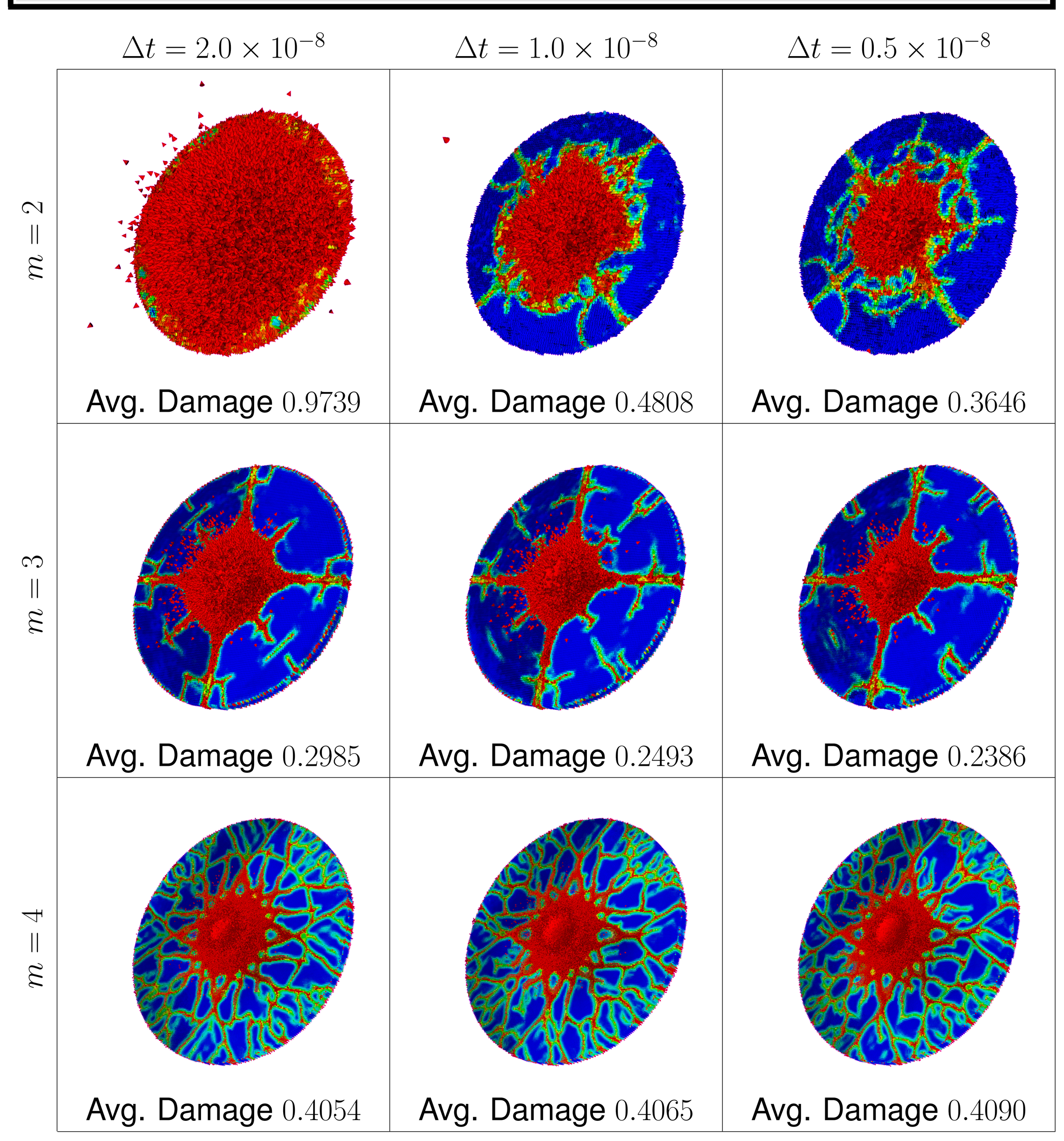
In the above equations, superscripts indicate the time step number during which a quantity is evaluated, and subscripts represent the node number.

4. Problem Setup

The impact of a target by a high speed projectile has become a benchmark problem for peridynamics. In this section, we use the impact problem as a prototype for investigating how meshing affects fracture simulations. The initial problem geometry consists of a high speed spherical projectile incident upon a cylindrical plate. The impactor has radius $r = 0.45\,\mathrm{cm}$. The target has radius $R = 3.75\,\mathrm{cm}$ and thickness $H = 0.30\,\mathrm{cm}$. The center of the projectile is displaced by a distance $d_0 = 0.18\,\mathrm{cm}$, which is slightly larger than $r + \delta$, from the top surface of the plate. As the simulation transpires, the impactor collides with the target with both sustaining damage.

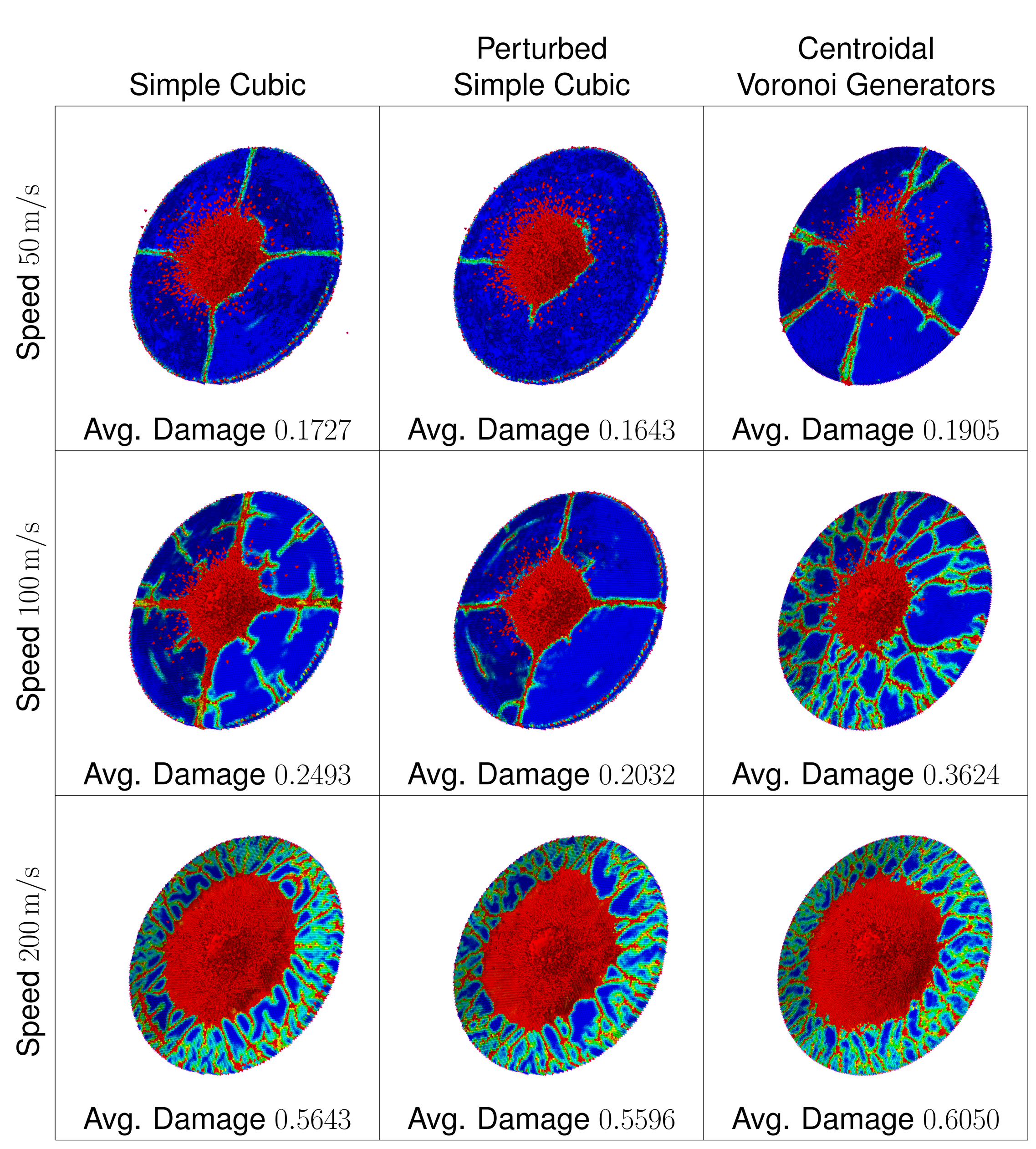


The bodies have the same composition, with mass density $\rho = 2200\,\mathrm{kg\,m^{-3}}$, horizon $\delta = 5 \times 10^{-03}\,\mathrm{m}$, critical stretch $s_0 = 5 \times 10^{-04}$, bonded modulus $c^b = 1.686 \times 10^{22}\,\mathrm{N\,m^{-4}}$, and non-bonded modulus $c^s = 15c^b$.



	$\Delta t = 2.0 \times 10^{-8}$	$\Delta t = 1.0 \times 10^{-8}$	$\Delta t = 0.5 \times 10^{-8}$
$m = 2$	Avg. Damage 0.9739	Avg. Damage 0.4808	Avg. Damage 0.3646
$m = 3$	Avg. Damage 0.2985	Avg. Damage 0.2493	Avg. Damage 0.2386
$m = 4$	Avg. Damage 0.4054	Avg. Damage 0.4065	Avg. Damage 0.4090

Table 1: *Effects of spatial and temporal refinement on damage patterns. All simulations use a simple cubic grid and an impact velocity of $100\,\mathrm{m/s}$.*

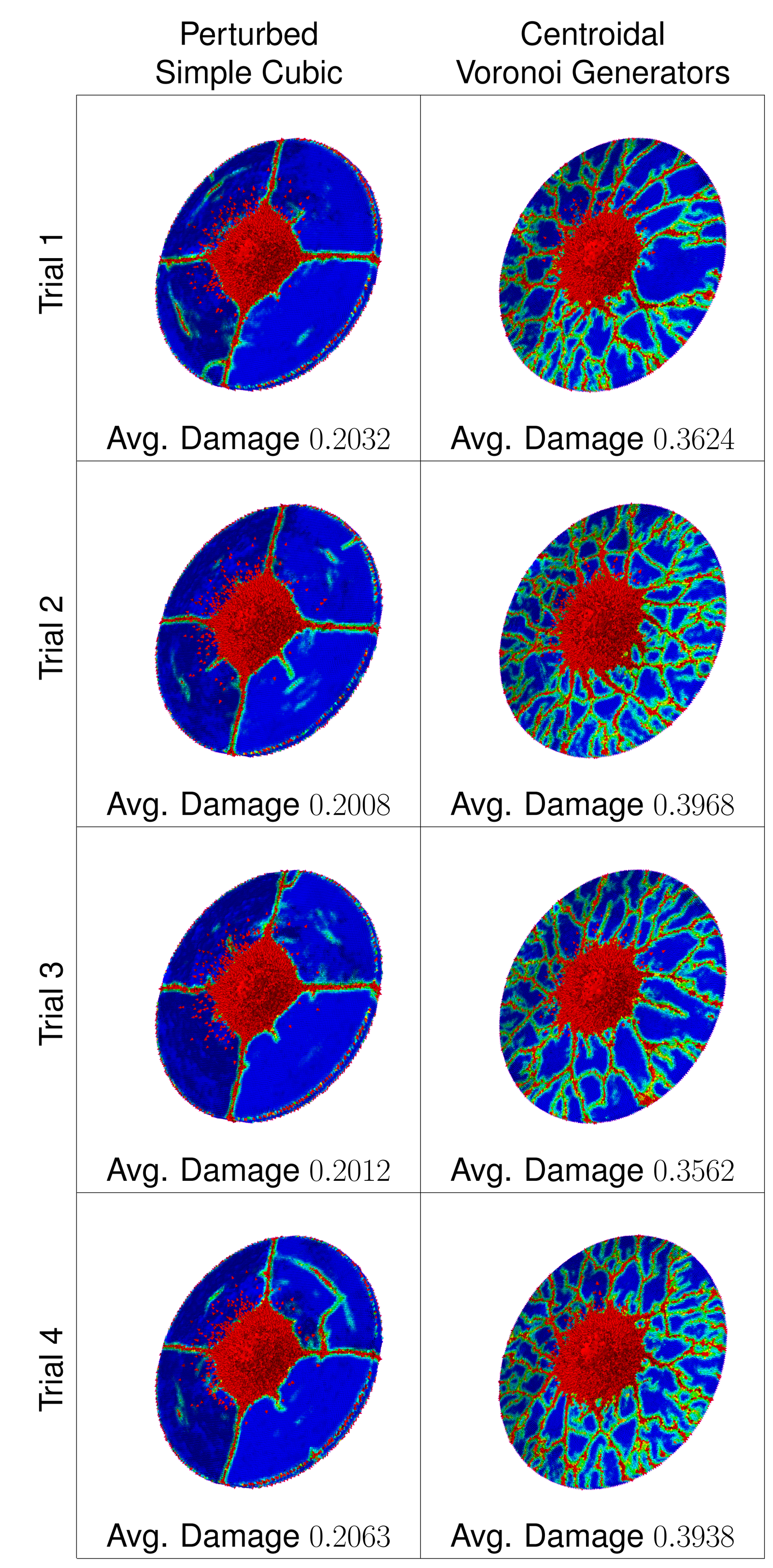


	Simple Cubic	Perturbed Simple Cubic	Centroidal Voronoi Generators
Speed $50\,\mathrm{m/s}$	Avg. Damage 0.1727	Avg. Damage 0.1643	Avg. Damage 0.1905
Speed $100\,\mathrm{m/s}$	Avg. Damage 0.2493	Avg. Damage 0.2032	Avg. Damage 0.3624
Speed $200\,\mathrm{m/s}$	Avg. Damage 0.5643	Avg. Damage 0.5596	Avg. Damage 0.6050

Table 2: *Effects of impactor speed on damage patterns. All simulations were carried out with the same number of quadrature points (corresponding to a simple cubic grid with $m = 3$) and time step $\Delta t = 10^{-8}$.*

5. Motivation

- The **simple cubic grid** is a straightforward generalization of a uniform 1-D mesh to multiple dimensions. The regularity of tensor product grids may not always be desirable, especially in fracture simulations where cracks have a tendency to follow symmetry lines in the mesh.
- A **uniform random perturbation** can be introduced to break symmetries in the simple cubic grid. Modifying the particle positions affects the accuracy of the quadrature scheme and introduces a source for additional computation errors.
- The generator points for a **centroidal Voronoi tessellation** (CVT) have previously been reported [4] to be high quality point sets for (local) meshless methods. CVT point distributions provide a more-faithful resolution of curved boundaries (avoiding the Cartesian staircase effect), and support adaptive refinement and non-uniform point densities.



	Perturbed Simple Cubic	Centroidal Voronoi Generators
Trial 1	Avg. Damage 0.2032	Avg. Damage 0.3624
Trial 2	Avg. Damage 0.2008	Avg. Damage 0.3968
Trial 3	Avg. Damage 0.2012	Avg. Damage 0.3562
Trial 4	Avg. Damage 0.2063	Avg. Damage 0.3938

Table 3: *Multiple realizations of the irregular grid types demonstrate the variety of fracture patterns that each supports. All simulations contain the same number of grid points (corresponding to a simple cubic grid with $m = 3$), use an impact velocity of $100\,\mathrm{m/s}$, and time step $\Delta t = 10^{-8}\,\mathrm{s}$.*

6. Summary & Conclusion

In this poster,

- we provided evidence that the time and space domains contained sufficient detail, then demonstrated how the computational mesh exerts influence on the outcome of the simulations at intermediate resolutions.
- the regularity of the cubic meshes was problematic, resulting in cracks that propagate along lines of symmetry in the mesh
- fracture patterns on the CVT generators contained complex branching patterns that compared favorably to the results obtained at higher resolutions.
- repeated simulations of the irregular grids displayed noticeably different crack paths, but represent qualitatively similar behaviors.

Peridynamic theory shows great promise in describing materials phenomena that include evolving discontinuities. To fully realize this promise it will be important to develop strategies that ensure the computational mesh exerts minimal interference on the outcome of a simulation.

References

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