Scalable Motion Segmentation using Swendsen-Wang Cuts

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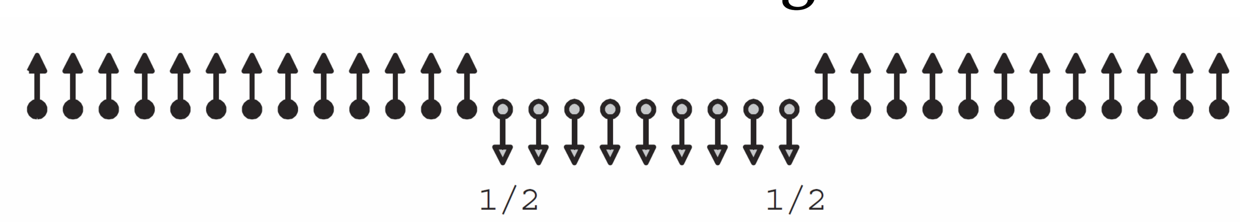
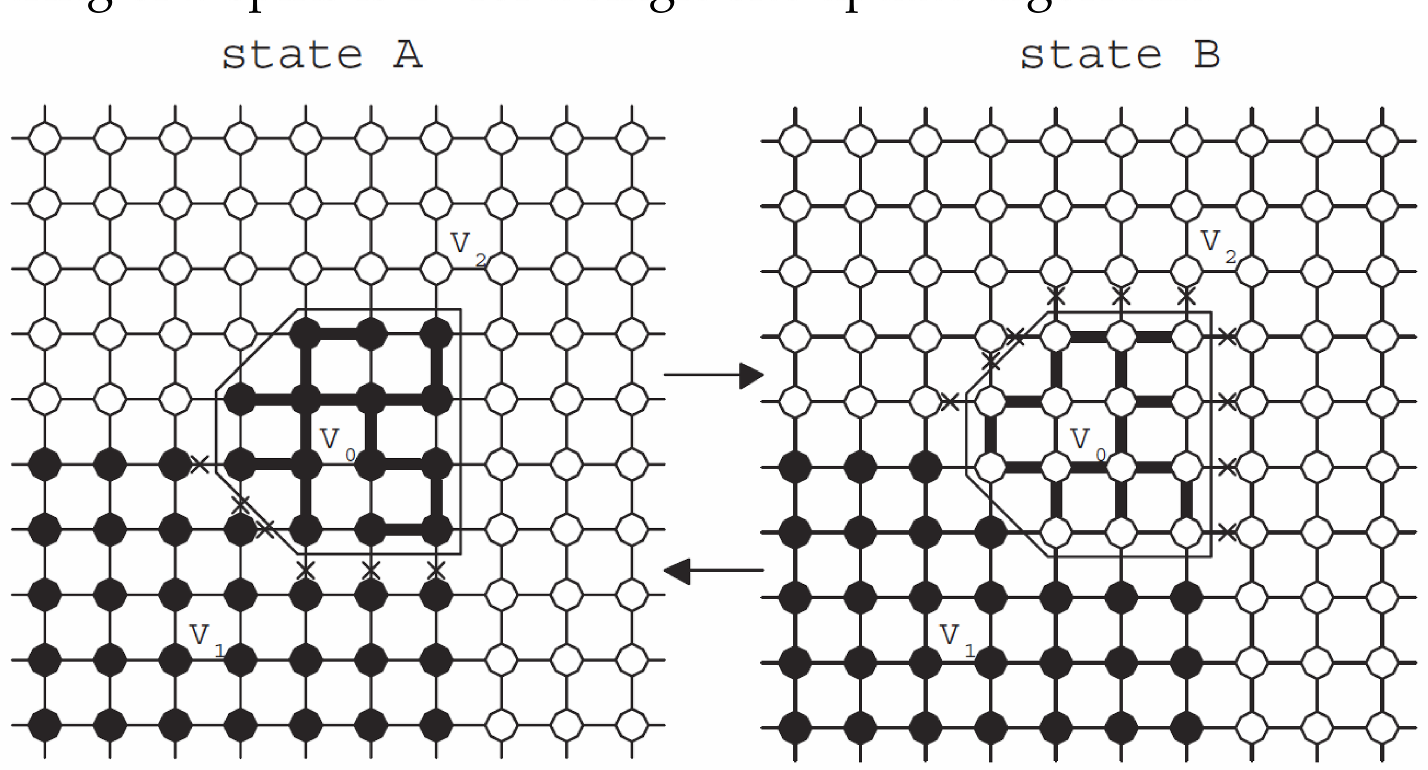
Overview

Task: cluster a set of tracked feature points based on their motions

Swendsen-Wang Cuts



In Ising and Potts model, the probability for flipping each spin from −1 to +1 is 1/2. Thus the expected number of steps to flip a string of k spins is 2^k for a single site update algorithm.



Swendsen-Wang method:

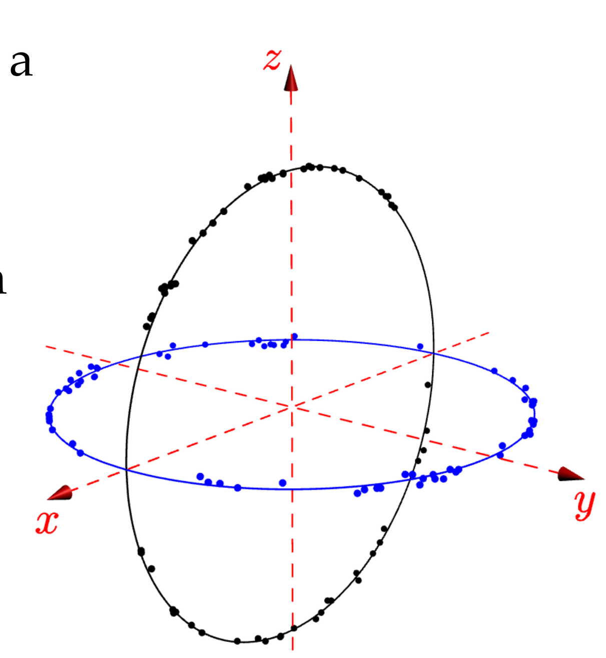
1. Turn edge "on" or "off" probabilistically.
2. Select a connected component V_0 of the resulting graph randomly.
3. Choose a label for V_0 with uniform probability.

Affinity Measure

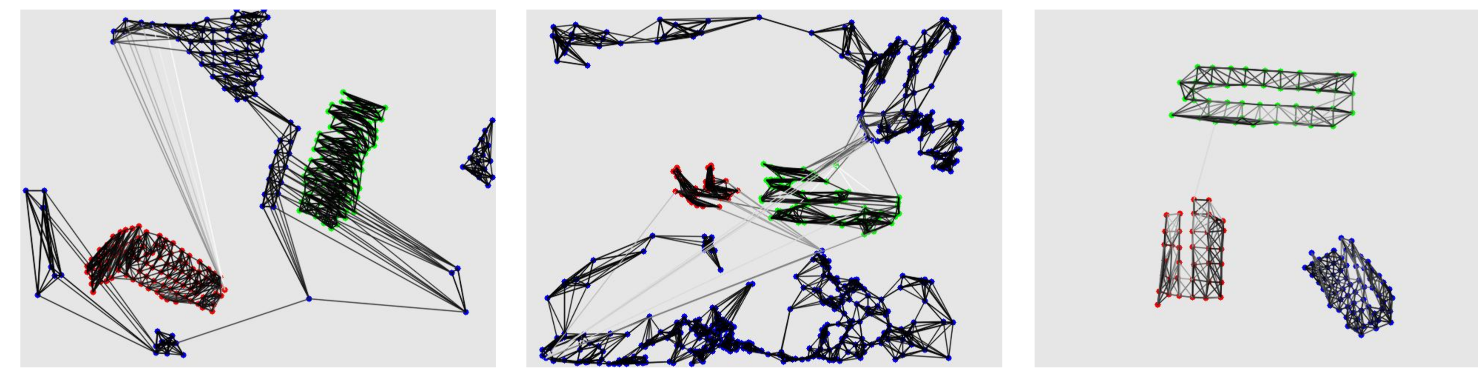
- The trajectories of feature points from a rigidly motion reside in an affine subspace of dimension at most 3.
- Affinity based on angular information

  $$A_{i,j} = e^{-m\theta_{i,j}/\bar{\theta}}$$

  where $\theta_{i,j}$ is the angle between two points x_i and x_j and $\bar{\theta}$ is the average.
- Overestimation could only happen at the intersections of subspaces.



- Build the adjacency graph by k nearest neighbors (NN).



Posterior Probability and Simulated Annealing

- The posterior Probability in a Bayesian framework:

  $$p(\pi) \propto \exp(-E_{data}(\pi) - E_{prior}(\pi))$$

- Data term: the sum of distances from points with label l to the affine subspace L_l which is fitted in a least square sense.

  $$E_{data}(\pi) = \sum_{l=1}^{M} \sum_{i,\pi(i)=l} d(x_i, L_l)$$

- Prior term encourages the points from the same connected components to stay in the same cluster.

  $$E_{prior}(\pi) = \sum_{<i,j>\in E, \pi(i)\neq\pi(j)} \rho A_{ij}$$

- Simulated annealing: $p(\pi) \rightarrow p(\pi)^{1/T}$.
- A faster annealing schedule:

  $$T_i = \frac{T_{\text{end}}}{\log(\frac{i}{N}(e - \exp(\frac{T_{\text{end}}}{T_{\text{start}}})) + \exp(\frac{T_{\text{end}}}{T_{\text{start}}}))}, i = 1, \ldots, N^{it}$$

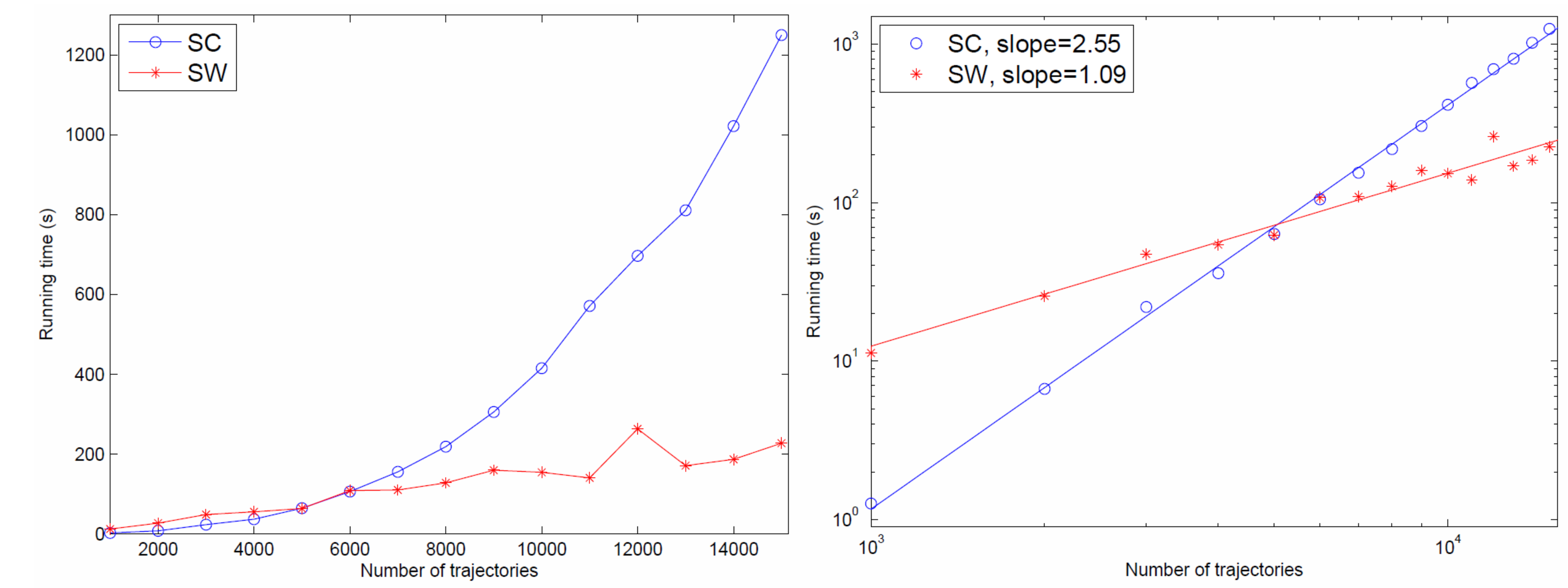
Full Algorithm

Input: N trajectories $(t_1, \ldots, t_N)$ from M motions
Dimension reduction: Project the trajectories to a D-dimensional space by truncated SVD, obtaining points $(x_1, \ldots, x_N)$.
Compute the affinity matrix A using the proposed measure.
Construct the adjacency graph G as a k-NN graph based on the affinity matrix A.
for $r = 1, \ldots, Q$ **do**
  for $i = 1, \ldots, T$ **do**
    1. Calculate the temperature t_i.
    2. Run the Swendsen-Wang cuts algorithm and record the final probability p_r.
  end for
end for
Output: M clusters with the smallest p_r.

Complexity Analysis

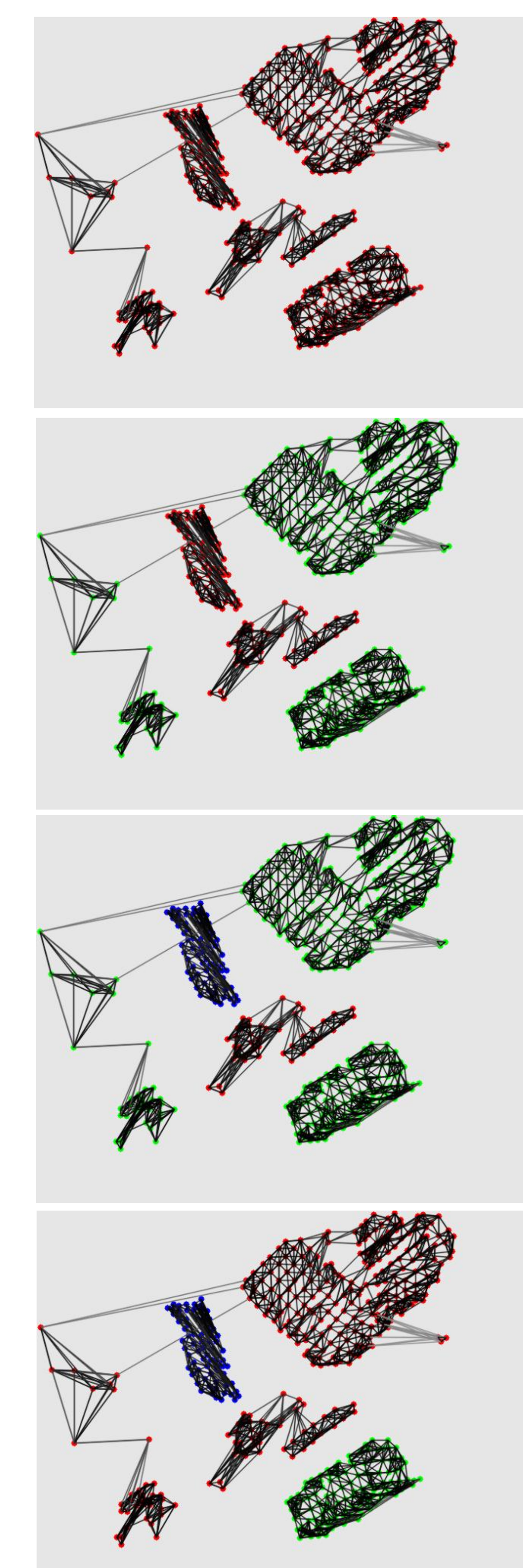
- The time complexity of the entire algorithm in terms of N trajectories is $O(N^2)$, comparing to $O(N^3)$ of spectral clustering.

- A simulated sequence of 3 motions, pick different number of trajectories for clustering and record the running time.



Experiments in the Hopkins 155 Dataset

Hopkins 155 is an extensive benchmark for testing feature based motion segmentation algorithms.



Clustering a sample sequence.

Method	ALC	SC	SSC	VC	SW
Checkerboard (2 motion)					
Average	1.55	0.85	1.12	0.67	1.90
Median	0.29	0.00	0.00	0.00	0.00
Traffic (2 motion)					
Average	1.59	0.90	0.02	0.99	1.53
Median	1.17	0.00	0.00	0.22	0.00
Articulated (2 motion)					
Average	10.70	1.71	0.62	2.94	1.76
Median	0.95	0.00	0.00	0.88	0.00
All (2 motion)					
Average	2.40	0.94	0.82	0.96	1.79
Median	0.43	0.00	0.00	0.00	0.00
Checkerboard (3 motion)					
Average	5.20	2.15	2.97	0.74	2.70
Median	0.67	0.47	0.27	0.21	0.58
Traffic (3 motion)					
Average	7.75	1.35	0.58	1.13	0.95
Median	0.49	0.19	0.00	0.21	0.93
Articulated (3 motion)					
Average	21.08	4.26	1.42	5.65	6.33
Median	21.08	4.26	0.00	5.65	6.33
All (3 motion)					
Average	6.69	2.11	2.45	1.10	2.56
Median	0.67	0.37	0.20	0.22	0.83
All sequences combined					
Average	3.37	1.20	1.24	0.99	1.92
Median	0.49	0.00	0.00	0.00	0.00

The misclassification rate.

Although SW performs slightly worse than some state-of-the-art algorithms based on spectral clustering in misclassification rate , it achieves a better scalability for large problems.