Investigating Vesicle-substrate Adhesion Using Two Phase Field Functions

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Adhesion of Vesicle to Substrate

Vesicle-substrate adhesion is a ubiquitous phenomenon in our world. Various Application of this kind of adhesion in bio-medical science happens in:

- target plasma
- stem cells
- leukocytes rolling

It is interesting and also necessary to understand the behavior of a cell membrane (phospholipid bilayer structured) interacting with a hard wall such as vessel wall.

Bending Energy and Contact Potential

The sharp interface model describing the vesicle substrate system is a combination of elastic bending energy of a membrane and contact potential energy between the membrane and the substrate:

$$E = \int_\Gamma K(H - c_0)^2 ds + \int_\Gamma W ds, \tag{1}$$

Where $H = \frac{k_1+k_2}{2}$ is the mean curvature of the membrane surface and $W = W(d(x))$ is the potential depending on the distance to the susbtrate.

Phase Field Model

Our phase field function has a tanh profile with the value outside the membrane being -1 and inside the membrane being +1. The width of the transitional layer simulating the membrane surface is controlled by the small parameter ϵ:

$$\Phi(x) = \tanh\left(\frac{d(x)}{\sqrt{2}\epsilon}\right) \tag{2}$$

Our phase field model is the variational problem:

$$\begin{aligned} \min E(\Phi_1, \Phi_2) = & \frac{k}{2\epsilon}\int_\Omega (\epsilon \triangle \Phi_1 \\ & + (\frac{1}{\epsilon} + c_0\sqrt{2})(1-\Phi_1^2))^2 dx \\ & + \frac{1}{2\epsilon}\int_\Omega (1-\Phi_1^2)(1-\Phi_2^2) dx \end{aligned} \tag{3}$$

With constraints of prescribed volume and surface area of the vesicle:

$$V(\Phi_1) = \int_\Omega \Phi_1 dx = \alpha, \tag{4}$$

and

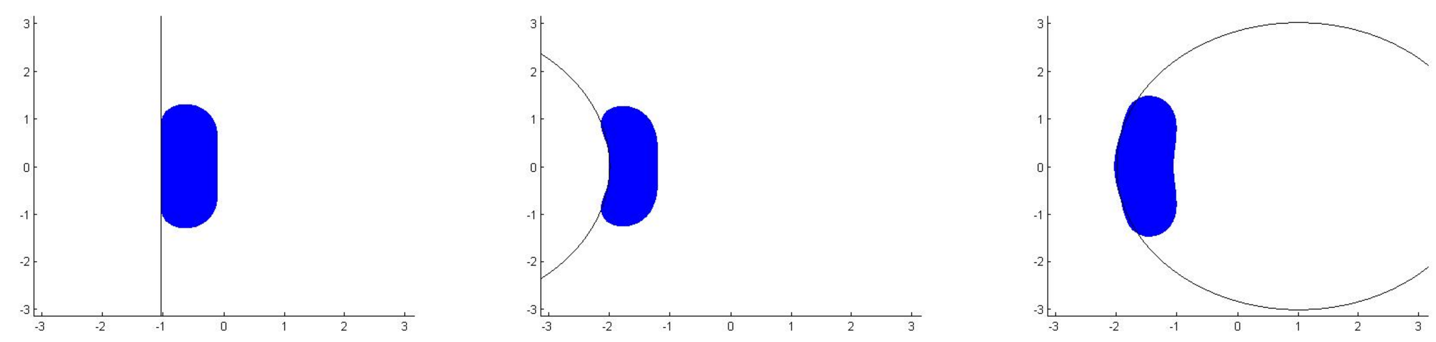
$$A(\Phi_1) = \int_\Omega \frac{\epsilon}{2}|\nabla \Phi_1|^2 + \frac{1}{4\epsilon}(\Phi_1^2 - 1)^2 dx = \beta. \tag{5}$$

Gradient Flow method is used to carried out the computational work on this problem:

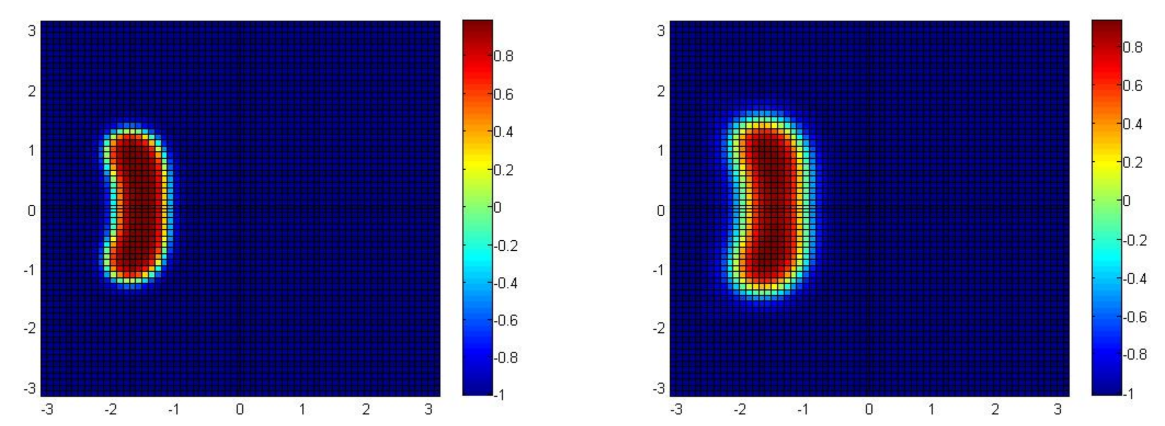
$$\frac{d\Phi_1}{dt} = -\gamma \frac{\delta E(\Phi_1, \Phi_2)}{\delta \Phi_1} \tag{6}$$

Numerical Results

Listed below are three adhesion situations due to different substrate shapes, the first one with a straight flat substrate, the second one with a convex cylindrical substrate and the third one with a concave cylindrical substrate.



Also the effect of transitional layer width is shown on a sliced view of vesicle, left figure with small width and right figure with large width.



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