On the Role of the Influence Function in the Peridynamic Theory

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Abstract

The influence function in the peridynamic theory is used to weight the contribution of all the bonds participating in the computation of volume-dependent properties. In this work [4], we use influence functions to establish relationships between bond-based and state-based peridynamic models. We also demonstrate how influence functions can be used to control nonlocal effects within a peridynamic model independently of the length scale set by the peridynamic horizon. We explore the effects of influence functions by studying wave propagation in simple one-dimensional models, and brittle fracture in three-dimensional models.

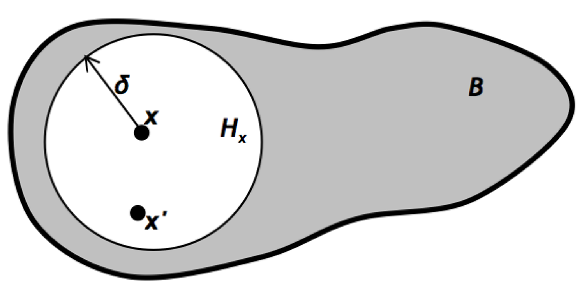
I. The Peridynamics Theory

State-based Peridynamics

In the state-based peridynamics (PD) theory, the deformation at a point depends collectively on all the points interacting with it. The PD equation of motion is [1]

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{\mathcal{H}_\mathbf{x}} \left\{ \underline{\mathbf{T}}[\mathbf{x},t]\langle \mathbf{x}'-\mathbf{x}\rangle - \underline{\mathbf{T}}[\mathbf{x}',t]\langle \mathbf{x}-\mathbf{x}'\rangle \right\} dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t), \tag{1}$$

with ρ the mass density, $\mathbf{u}$ the displacement field, and $\mathbf{b}$ a body force. A point $\mathbf{x}$ interacts with all the points $\mathbf{x}'$ within the neighborhood $\mathcal{H}_\mathbf{x}$, assumed to be a spherical region of radius $\delta > 0$ centered at $\mathbf{x}$; δ is called the *horizon*. The vector force state $\underline{\mathbf{T}}[\mathbf{x},t]\langle\cdot\rangle$ is a mapping, having units of force per volume squared, of the vector $\mathbf{x}'-\mathbf{x}$ to the force vector state field. We consider only models that can be written as $\underline{\mathbf{T}} = \underline{t}\,\underline{\mathbf{M}}$, with $\underline{t}$ a *scalar force state* and $\underline{\mathbf{M}}$ the *deformed direction vector state*.



Influence functions

An influence function is a nonnegative scalar state $\underline{\omega}$ defined on $\mathcal{H}_\mathbf{x}$. If an influence function $\underline{\omega}$ depends only upon the scalar $|\mathbf{x}'-\mathbf{x}|$, then $\underline{\omega}$ is a spherical influence function. Influence functions are used in the construction of peridynamic constitutive models. They were suggested as a selection mechanism to determine which bonds participate in the calculation of the force state and other related quantities [1].

We show that influence functions can be used to connect families of peridynamic constitutive models, and to moderate the strength of nonlocal interactions

Peridynamic Force States Derived from Pairwise Potentials

Conservative forces derive from a potential. If we assume a force state derives from a pairwise potential (as opposed to a multibody potential), it can be written as using the scalar force state field

$$\underline{t}[\mathbf{x},t]\langle \mathbf{x}'-\mathbf{x}\rangle = \frac{1}{2} f\left(\mathbf{u}(\mathbf{x}',t)-\mathbf{u}(\mathbf{x},t), \mathbf{x}'-\mathbf{x}\right), \tag{2}$$

with f a scalar-valued function, having units of energy per volume squared. Then, (1) reduces to

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{\mathcal{H}_\mathbf{x}} \mathbf{f}\left(\mathbf{u}(\mathbf{x}',t)-\mathbf{u}(\mathbf{x},t), \mathbf{x}'-\mathbf{x}\right) dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t). \tag{3}$$

Equation (3) is the equation of motion corresponding to the *bond-based* PD theory presented in [2].

II. Peridynamic Constitutive Models and Their Relationships

Two of the most commonly used peridynamic constitutive models are the *linear peridynamic solid* (LPS) model [1] and the *prototype microelastic brittle* (PMB) model [3].

Linear Peridynamic Solid (LPS)

$$\underline{t} = \frac{3K\theta}{m}\underline{\omega}\,\underline{x} + \alpha\underline{\omega}\,\underline{e}^{\mathrm{d}}, \tag{4}$$

with $\underline{x}$ the reference position scalar state, m the weighted volume, $\underline{e}$ the extension scalar state, $\alpha = \frac{15G}{m}$, G the shear modulus, K the bulk modulus, θ the dilatation, and $\underline{e}^{\mathrm{d}}$ the deviatoric part of the extension.

For the specific spherical influence function

$$\underline{\omega}\langle \mathbf{x}'-\mathbf{x}\rangle = 1/|\mathbf{x}'-\mathbf{x}| \tag{5}$$

and a Poisson ratio $\nu = 1/4$, we obtain, by using relation (2), the bond-based PMB model (6).

Prototype Microelastic Brittle (PMB)

$$f = c\,s, \tag{6}$$

with $c = 18K/\pi\delta^4$, and s the stretch.

Utilizing the spherical influence function (5), we have shown that the bond-based PMB model is a special case of the state-based LPS model, with Poisson ratio $\nu = 1/4$

III. Generalized PMB Models

We may derive generalized PMB (GPMB) models by writing the influence function in (4) as

$$\underline{\omega}\langle \mathbf{x}'-\mathbf{x}\rangle = \omega_g\left(|\mathbf{x}'-\mathbf{x}|\right)|\mathbf{x}'-\mathbf{x}|^{-1} \tag{7}$$

for a Poisson ratio $\nu = 1/4$, with $\omega_g\left(|\mathbf{x}'-\mathbf{x}|\right)$ a general spherical function. The resulting pairwise force function is

$$f = c_g\,s\,\omega_g\left(|\mathbf{x}'-\mathbf{x}|\right). \tag{8}$$

We define c_g by equating the macroelastic energy density of the GPMB model under an isotropic extension, to the strain energy density in the classical theory of elasticity for the same material and the same deformation.

There exist many GPMB models of the form (8), including the PMB model (6), all of which are special cases of the LPS model (4)

IV. Numerical Examples

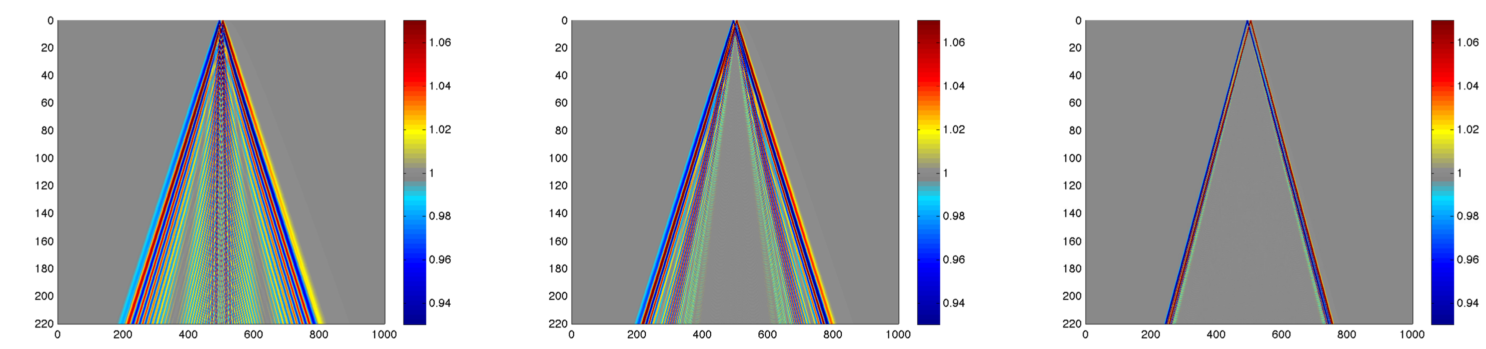
One-dimensional example

We explore the role of the influence function through a one-dimensional example, demonstrating that an increasingly localized behavior can be obtained from a nonlocal model by moderating the strength of the nonlocal interactions. We define

$$\Psi_p\left(|\mathbf{x}'-\mathbf{x}|\right) := \left(\frac{1}{|\mathbf{x}'-\mathbf{x}|+\epsilon}\right)^p, \tag{9}$$

with $\epsilon > 0$ a softening length. The family of p-dependent spherical functions (9) serve as a weighting of the strength of the bonds integrated within (3). Inserting this specific family of functions into (7) defines an influence function $\underline{\omega}\langle \mathbf{x}'-\mathbf{x}\rangle$.

We present numerical results comparing the density evolution of a smooth pulse in a one-dimensional system, for different values of p. We implement the spherical function (9) in model (8). Dispersion is numerically manifested as broadening of the lines. We choose a domain $\mathcal{B} = [0, 1000]$ with 4001 equally spaced nodes. Furthermore, $\delta = 5$, $K = 1$, $\rho = 1$, $b = 0$, $\epsilon = 1$, and $\Delta t = 0.56$. The $p = 10$ case is the least dispersive.

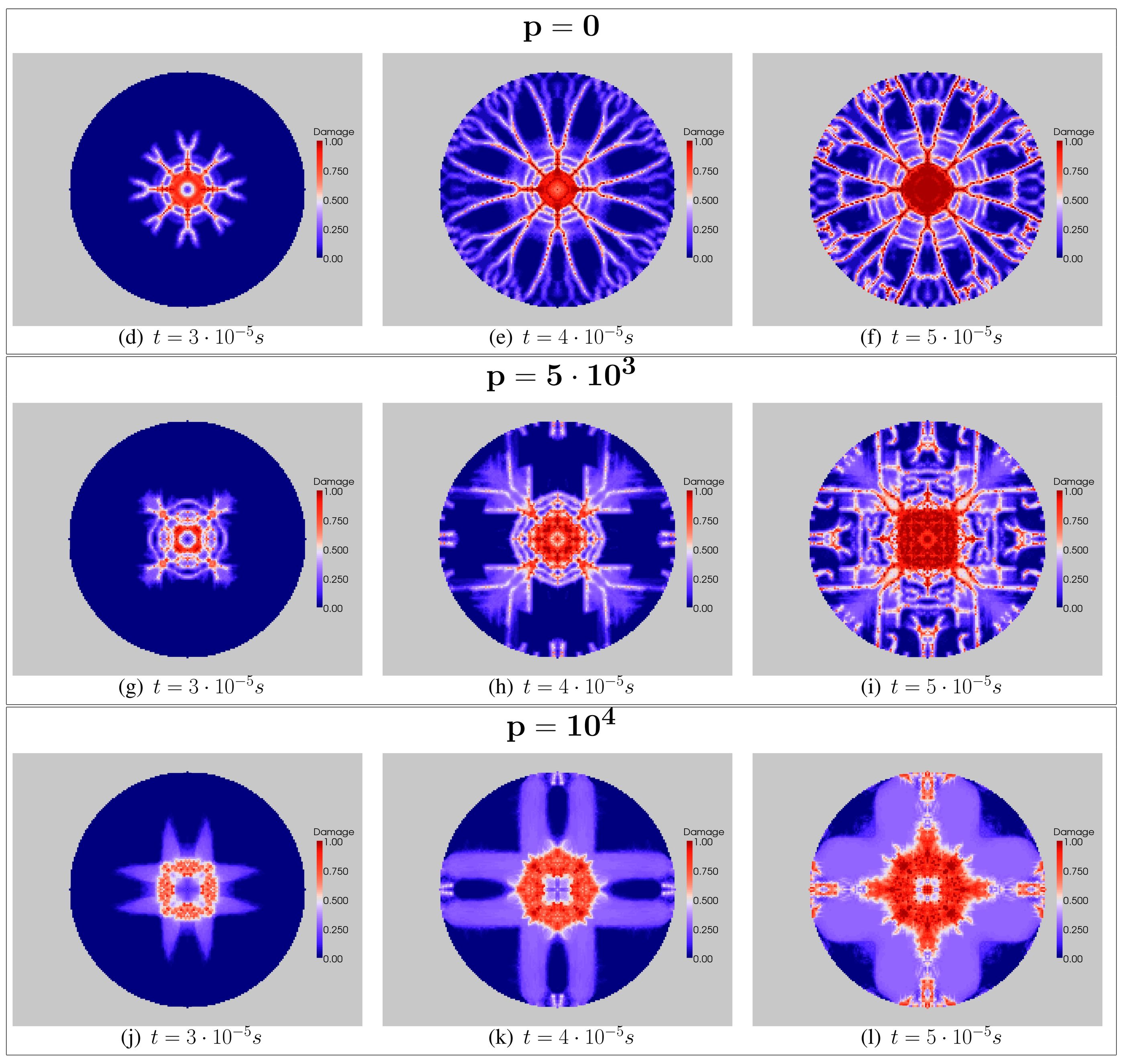


(a) $p = 1$ (b) $p = 3$ (c) $p = 10$

Three-dimensional example

To investigate the effect of the influence function on the evolution of fracture patterns, we utilize the PDLAMMPS code, and assume bonds break if they are stretched too far. Following [3], we simulate the impact of a hard sphere on a brittle target. The target is a thin disc of diameter 74 mm and 2.5 mm thick, having bulk modulus $K = 14.9$ GPa, density $\rho = 2200\,\mathrm{kg/m^3}$, and $\epsilon = 1\,\mathrm{m}$ in (9). The spherical projectile has a radius of 10 mm and a velocity of 100 m/s. The target has a total of about 103,000 particles, and the horizon is chosen as $\delta = 1.5$ mm. The time step is chosen as $\Delta t = 10^{-9}$ s, which is CFL-stable.

We present top-down views of the target for various time steps, for different values of p, with the projectile removed so as to not obscure the target. We observe that for larger values of p cracks grow essentially along the axes of the mesh, whereas the cracks for $p = 0$ appear to be independent of the mesh. This is consistent with the hypothesis that large values of p lead to models dominated by local and nearly-local interactions. The colors represent damage, which is computed as one minus the number of unbroken bonds in the current configuration divided by the number of unbroken bonds in the reference configuration.



(d) $t = 3\cdot 10^{-5}s$ (e) $t = 4\cdot 10^{-5}s$ (f) $t = 5\cdot 10^{-5}s$
(g) $t = 3\cdot 10^{-5}s$ (h) $t = 4\cdot 10^{-5}s$ (i) $t = 5\cdot 10^{-5}s$
(j) $t = 3\cdot 10^{-5}s$ (k) $t = 4\cdot 10^{-5}s$ (l) $t = 5\cdot 10^{-5}s$

V. Conclusions

To explore the role of influence functions within the peridynamic theory, we began by demonstrating that the bond-based PMB model (6), the most popular bond-based peridynamic model, is a special case of the state-based LPS model (4), for Poisson ratio $\nu = 1/4$ and influence function (5). We also showed that there exists a related family of generalized PMB models that can be derived by selecting other influence functions. Using one-dimensional examples, we showed that influence functions can be used to control nonlocal effects within a peridynamic model independently of the length scale set by the peridynamic horizon, δ. Lastly, we investigated the effect specific influence functions have on dynamic fracture simulations, reporting results for the impact of a hard sphere on brittle targets.

References

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